Thesis accepted for the Ph. D. Degree of Nagpur University.

Time, Space and Motion

A Logical Analysis with Special Reference to Psychology.

By

N. R. W. PANDE
B. LITT. (OXON), PH. D.,
KAVYATEERTH, SHASTRI.
Principal Scientific Officer,
Government of India.

Printed at
NAGPUR VIDYAPEETH MUDRANALAYA,
NAGPUR.
1969

Published by—

S. V. Bhagwat,
Registrar,
Nagpur University,
Nagpur.

Price : Rs. 15

FIRST EDITION

Printed by—

S. L. Turile,
Superintendent,
Nagpur Vidyapeeth
Mudranalaya,
Nagpur.

Foreword

I am glad in presenting this work on 'Time, Space and Motion' by Dr. N. R. Warhadpande. He has discussed many time-honoured and fundamental problems in Logic and Logico-mathematics and discussed their bearing on Psychology. His treatment, therefore, has not remained of a purely theoretical nature but may prove to be of value to sciences like Psychometry and Experimental Psychology. The treatment of (1) Fundamentals of Measurement and their Applicability to Psychology, (2) Perception of Motion and (3) Life, Motion and Predictability can be specifically mentioned in this connection.

I hope experts in the field of Logic, Mathematics and Psychology will take notice of Dr. Warhadpande's work and subject it to critical examination.

V. B. Kolte
Vice-Chancellor,
Nagpur University.

Nagpur:
The 10th September, 1969.

पूर्णमदः पूर्णमिदं पूर्णात्पूर्णमुदच्यते
पूर्णस्य पूर्णमादाय पूर्णमेवावशिष्यते

—बृहदारण्यकोपनिषत्

This is infinite, so is that, the infinite has come out of the infinite. When the infinite is subtracted from the infinite the remainder is still infinite.

—Upanishat

Contents

I

The Scope of the Present Inquiry

It is necessary to discuss at length what this work is about as the title may suggest many different things. The work originated in a consideration of experiments on perception of motion on the one hand and Zeno's denial of the very existence of motion on the other. Superficially, the first appears to be a domain of Psychology whereas the second that of Logic and Philosophy. But a fuller consideration showed that the second also has a vital bearing on the fundamental problem of the young science of Psychology, *viz.*- the application of measurement. Zeno has touched the very roots of the concept of time, space and motion and the nature of these is bound up with the nature of the number continuum and the basic principles of measurement.

The method followed in the forthcoming pages is primarily that of logical analysis of the fundamental notions as they find expression in ordinary language. In sciences such as Pure Mathematics and Formal Logic, an author may coin new terms and give them arbitrary meanings which he proposes to use consistently. But this is not the way of Philosophy. When a philosopher analyses the word 'good' he takes the word from ordinary usage and his endeavour is to find out which of its definitions, if any, fits all the contexts

in which it is used, and if no one definition fits all the contexts, whether its different meanings are related to each other, and if so in what way, whether they logically imply each other and whether finally the word stands for something real or for nothing.

It should not be supposed that this last endeavour of philosophy, namely that of finding out whether the word stands for something real or not, is a sort of fact finding programme, as in empirical sciences, where a Zoologist may try to find out how many teeth a crocodile has. In order to find out how many teeth a crocodile has, he has to undertake special observation and when this is undertaken, there is generally no disagreement about the findings. But the question of reality is not settled in this way in Philosophy. Two philosophers, who both see a table, may not yet agree whether it is real or not, or even on what exactly is observed when it is said that a table is observed. The question of reality in philosophy is, therefore, a logical question, which cannot be settled by more accurate observations. It can be settled only by analysis of the meanings of the words true, real, etc.

Some readers may feel that this sort of inquiry is merely "metaphysical" and too old-fashioned to be indulged in a work that claims to be scientific. I do not share this feeling. As will be clear from the sequel, fundamental logical notions are always involved in any scientific methodology. In deciding whether an instrument of observation reveals the truth or falsifies the data, we must have a clear grasp of the distinction between truth and illusion. Similarly, the basic principles involving measurement may come instinctively in the realm of physics, but when we wish to extend their use to new fields like psychology, a mere instinctive awareness is not enough. A clear logical formulation is necessary for answering criticisms and developing new tools of measurement.

The present discussion concerns itself with the analysis of the meanings of the words used in describing time, space and motion in different contexts and laying bare the logical notions involved.

The present work does not concern itself with questions which have no more than merely a historical value. I propose to confine myself to problems which are still living and will not elaborate points which nobody would take seriously today, nor do I wish to enter into controversies like "what Kant exactly meant"? While discussing the ideas of different writers, I have taken into account only the most obvious interpretation of their words. I expect that even those who have some special interpretations of the words of great writers to offer, will still agree that the interpretations which I have suggested are valuable.

In the same way, no attempt will be made to trace the historical origin of an argument or to establish priority among the writers who have advanced it. Neither shall I endeavour to fix the original form in which it was first advanced. I will discuss arguments mostly in the form in which they are usually quoted and discussed or the form in which they are logically valuable.

Just as the inquiry is not historical, it is also not intended to be encyclopaedic. I do not intend to discuss every topic that can be covered by the title, but only those on which I have something to say either by way of comments on the ideas of other writers or by way of breaking new ground.

II

The Definition of Motion

Motion is generally defined in the following way :—*A* moves, means that *A* occupies a particular place at one time and another place at a subsequent time.

This definition has been criticised on the following grounds :

(1) From the fact that *A* was at p_1 at one time t_1 and at p_2 at a subsequent time t_2, we infer that it has moved from p_1 to p_2. But this does not entitle us to say that this fact constitutes the definition of motion. It will be agreed that we can see motion, but a man who has seen *A* at p_1 and then at p_2, cannot be said to have seen motion. Motion has occurred in between the two places and the two times, and not *at* them.

The answer to this objection is that, the seeing of motion implies the seeing of all the places occupied by *A* and not merely the first and the last. A man who has merely seen *A* at p_1 and p_2 cannot be said to have seen it moving because he has not seen the places in between p_1 and p_2 which were occupied by *A*, at times in between t_1 and t_2. If he has seen *A* at all the positions it has occupied, can it still be said that he has not seen the motion of *A* ?

Some people make a capital out of the phrase "positions occupied" and say that a "position" is a state of rest and states

of rest however many cannot make motion. It is necessary, therefore, to define what is rest and to explain whether *A* can be at a place without being at rest there. A tentative definition of rest would involve the conception of an instant. An instant is indivisible and though it is in some sense a part of time, it has no duration. Now anything that is at rest is at a place during some time. If, therefore, it is only there for an instant we cannot say that it was at rest, because it was not at rest during any time. Rest would require being at a place for more than one instant.

Some people find the notion of an " instant " which has no temporal dimensions, extremely incomprehensible or even self-contradictory. This notion and the corresponding notion of a " point " will be discussed in a separate Chapter (IX). For the present it must be admitted that if instants are disallowed, it will have to be said that a moving object stays at the various places occupied by it for an unobservably short time and therefore motion is nothing but a series of static positions, if the definition under discussion is accepted.

The obvious objection to this is that a person who observes the successive positions of a moving object, for example the hour-hand of a clock, does not yet observe its motion, and therefore motion is something over and above mere successive positions.

This objection has an equally obvious answer, *viz.*- that something over and above mere existence is needed for anything to be observable. A book in microfilm cannot be read by the naked eye, but the letters in it do not differ from ordinary letters except with regard to size. A letter in order to be observable must not be too big nor too small. Similarly motion in order to be observable must not be too slow nor too fast. Conditions of observation therefore should not be confused with the conditions of the constitution of what is observed.

Another objection can be raised on Bergsonian lines, to the effect that successive positions may suffice to create an

illusion of motion, as in a cinematograph, but not real motion. If successive positions were the very essence of motion, it would not be meaningful to say that motion seen on the screen is not real.

In what sense then is this motion not real ?

One obvious reason for regarding this motion as unreal is that we know that between two consecutive positions of the picture on the screen, a finite amount of time elapses, and during this time the projected picture does not exist. In real motion, it is supposed that at all times the moving object exists somewhere.

A Gestaltist may here argue that, on analysis we may not find anything in motion except successive positions of an object but this will only suffice to show that motion is a whole composed of these positions, and since a whole is something over and above its parts, motion is something more than these successive positions. This doctrine of whole and part must therefore be examined.

There are various senses in which a whole is more than all its parts together. (1) If the word part is used in the sense of an element and does not refer to the relations obtaining between different elements, it is obvious that a mere list of such parts does not describe the whole, any more than a mere list of cities, mountains and rivers, etc., can serve as a correct description or a map of a country. A map must also show how these are situated in relation to each other. If we mean by parts the elements as well as the inter-relations of these elements, it will be difficult to maintain that we have not correctly described the whole by mentioning all their parts. Similarly a statement of all the positions of A and their temporal and spatial relations can be said to describe the motion of A correctly.

(2) There is another sense in which the whole is different from all its parts together. This is seen in the case of water. By mentioning hydrogen, oxygen and the proportion in which they form water, we cannot be said to have correctly described

water. Water is quite different from these parts and their interrelations.

In this particular case it is doubtful whether we are justified in regarding hydrogen etc., as " parts " of water. Hydrogen as hydrogen is not present in water, the only justification for supposing it to be present is that we use it in the composition of water and we can get it back from water by decomposition. Beyond this, there is no positive evidence that it was in existence during the time intervening between the composition and decomposition of water. The assumption of continuity of the existence of hydrogen in water is based on a false analogy with money put in the money bag. It exists unchanged in the bag after it is put there and before it is taken out. It can be proved by suitable tests that a rupee does not change its identity as a rupee when it is in the bag. Its shape, weight, characteristic, metallic sound, etc., can be ascertained while it is in the bag. Hydrogen can be properly regarded as a part of water, if it can be similarly proved that it does not lose its identity as hydrogen by entering into the formation of water.

Instead of speaking of hydrogen as a part of water, it will be more correct to speak of the attributes of water as parts of it. If we enumerate all the properties of water and the ways in which they are interrelated, it will be difficult to say that we have not correctly desribed water, and that water is something over and above these parts. Now the positions of A and their interrelations, do not lose their identity as positions and relations by being parts of the motion of A. The case therefore is not similar to the case of H_2O and water.

(3) The eternal problem of colour and the wave-length with which it is associated may be raised here. By enumerating all the physical and physiological antecedents of a colour sensation, such as wave-lengths and nerve-excitations, we cannot correctly describe the colour. The colour is *sui generis* and over and above all these factors which may be regarded as

parts of the colour-sensation. Motion, it may be said is similarly *sui generis*, the successive positions of a moving object, may be factors in the perception of motion, but it cannot be said that they *are* motion.

A little reflection will show that the case of motion is not analogous to the case of colour. A man who sees colour, does not see the wave-length. Human beings have seen colours ever since the dawn of history, but wave-lengths were discovered only after the dawn of Physics. On the other hand, a man who sees motion *does* see the successive positions of a moving object. It does not make sense to say that I have seen *A* moving, but I have not seen it occupying different positions at different times.

III

The Relativity of Motion

In the last Chapter we defined motion in terms of positions. But position is generally defined in terms of distance, which is a relation between two objects. Instead of defining motion therefore in terms of differences in position, we could define it in terms of differences in distance.

Such a definition at once introduces the conception of the relativity of motion. If the motion of *A* consists in its being at different distances from other objects at different times, it can be said to be moving only in relation to the objects from which it changes its distance, but not in relation to the objects from which it does not change its distance. Motion no longer remains an absolute property of an object but depends on its relations with other objects. This raises the famous difficulty that if the distance between *A* and *B* is diminishing, how can we know whether *A* is moving towards *B* or *B* is moving towards *A* or both are moving towards each other ?

Eddington says that it is as correct to say that the station moves to the train as to say that the train moves to the station[16] (p. 132). Let us see what are the implications of this.

If we say that the station moves to the train *A*, we shall have to assume that the whole earth is moving in relation to the

train in the direction opposite to that of the supposed motion of the train. If now a train *B* is approaching the station from the opposite direction, and we want still to maintain that the station is moving, we shall have to say that the station and the train *A* together are moving to the train *B*, just as a man walking on the boat towards the back of the ship and the back of the ship itself are together moving towards the shore. The man corresponds to the station, the back of the ship to the train *A*, and the shore to the train *B*. Similarly in the case of a third train coming to the station from a direction at right angles to the line joining the trains *A* and *B*, we shall have to suppose further that the trains *A*, *B* and the station are as a whole moving towards the train *C*. If a fourth train *D* is coming from the direction opposite to the train *C*, we shall have to assume *A*, *B*, *C* and the station as a whole moving towards the train *D*.

It is necessary to explain what is meant by moving as a whole. We said that the man walking to the back of the ship and the back of the ship are together moving to the shore because the man and the back of the ship are in the ship and the ship is moving to the shore. In the case of the third train *C*, we have to assume the shore (*i. e.* train *B*) the water and the ship as all contained in a bigger ship which is moving towards the shore (*i. e.* train *C*) of a bigger expanse of water. This second ship together with the bigger expanse of water and its shore are to be imagined again as contained in a still bigger ship which is moving towards the shore (*i. e.* train *D*) of a still bigger expanse of water. So the phrase " a class of objects move as a whole " means that it is spatially included in a bigger object which itself moves. Whatever is at rest in relation to this bigger object is said to be stationary within the object.

Spatial inclusion can be more precisely stated. An object *A* can be said to be spatially included in a class of objects *B, C, D* — — — if it is always *between* objects belonging

to the class, however, much it changes position. If *B, C, D* are stationary in relation to one another and *A* changes its position in relation to them it is supposed that *A* is moving and not *B, C, D*.....

The mariner's compass is spatially included in the ship. The ship can be defined as a class of objects such as its cabins, walls, etc., which are stationary in relation to one another. But still we say that it is the ship which changes its direction and not the compass needle, because the ship itself is included in the earth and the direction of the needle is stationary in relation to the earth. If there is a class of objects which are stationary in relation to each other and include all other objects, anything stationary in relation to this class can be said to be at absolute rest. The earth includes all other objects on the earth, such as trains and stations. The station being at rest in relation to the earth (*i. e.* a class of objects like mountains and rivers stationary in relation to each other) it is regarded as at rest, when we are not speaking of phenomena outside the earth.

When motion is perceived, an object which spatially includes appears as stationary, in relation to the object that is spatially included. This can be demonstrated experimentally. In one such experiment ... " one of the objects was a point ... and the other a contour rectangle ... The point was inside the rectangle. The speed of objective motion (whether of the point, or of the retangle) was always the same. In all cases, the unmoving object was the one fixated. There were ten observers. In all cases ... the strongest movement was that observed in the point, and not in the rectangle. Ordinarily (*e. g.* with two points) it is the fixated object which is seen to move; here the fixated rectangle never moved "[17] (p. 164).

The example of ships within ships which are necessary in order to maintain that it is the station and not the trains, which always moves, indicates that the attribution of motion and rest is verifiable in such a case and the two statements (1) the train moves to the station and (2) the station moves

to the train are not equivalent. Since no objects corresponding to the postulated ships can be found, the movement of the station can be proved to be false.

This of course does not imply "absolute motion or rest". As already explained, the station is at rest only in relation to the earth. Eddington's example serves to emphasize this fact and is to be interpreted accordingly.

Some people advance very shallow arguments in order to prove absolute motion. In his controversy with Descrates, Henry More makes a jesting allusion..... 'When I am quietly seated, and another going a thousand spaces away, is flushed with fatigue, it is certainly he who moves and I who am at rest'[4.1] (p. 255). An analogy can at once show the hollowness of this reasoning. A man painfully pulling a bucket of water from a deep well is fatigued, but the bucket is "quietly seated" like Henry More. He should, therefore, infer that the bucket is at rest and the man is moving towards it ! Fatigue can be no evidence of motion. When *A* and *B* are moving relatively to each other, the resulting fatigue in *A* may be caused because he is pulling or pushing *B* and not because he is himself moving.

Newton's experiment with the bucket similarly does not prove absolute motion. In the experiment (1) initially the bucket alone rotates and the water stays level, (2) gradually the water picks up the rotation and begins to rise on the walls of the bucket, (3) this rising continues even after the bucket comes to a stop. In stage 1, the water is moving in relation to the bucket, but it shows no effect of this motion. In the third stage, on the other hand, it is moving in relation to the bucket and shows the effect of the movement. In the second stage, it is at rest in relation to the bucket and shows the effect of rotatory movement. Motion in relation to the bucket is therefore irrelevant to the behaviour of the water. But this does not prove that the rising of the water is caused by the "absolute rotation" of the water. As rightly pointed out by Prof. Mach,

"Newton's experiment only shows us that the rotation of the water relative to the sides of the bucket occasions no perceptible centrifugal forces, but that such forces are occasioned when the water rotates *relatively to the masses of the earth and the other heavenly bodies*"[57] (p. 75).

Another famous argument for absolute motion is that from Foucoult's Pendulum. "We swing a large pendulum and we note the plane in which it begins to swing. We will suppose that we make a chalk mark on the floor in the line where this plane cuts it. As time goes on, if we draw lines on the same principle, we shall find that they make angles with each other"[7.3] (p. 294). Thus even if we confine ourselves to phenomena on the earth, we can infer its rotation from these changes. This shows, it is argued, that the rotatory movement of the earth is not merely a movement relative to something other than the earth.

As Broad has pointed out, the argument derives its force from the assumption, "..... that the plane in which the pendulum swings is fixed in absolute space"[7.3] (p. 294). It is on this assumption that the change in the lines corresponding to that plane is ascribed to the rotation of the earth.

Since all motion is relative, if the distance between *A* and *B* is constant we cannot say that they are moving in relation to each other. It might appear therefore that "movement around a centre " is not possible. When an object moves on a circle, it maintains a constant distance from the centre of the circle. The centre of the circle is a point and strictly speaking nothing can move around a point. We can move around a man because we can stand in front of him, then go to his left, then to his back, then to his right and then come back to the front again. But the point has no front and back, right and left.

Nevertheless there is a sense in which one can move around a point. If we draw lines through the point, an object supposed to be moving around the point changes its distance from all the points on these lines, excepting from the point in

question. The object therefore derivatively moves around the point, because it moves in relation to all the points around it.

When two objects are said to change their position in relation to each other without changing their distance, what is really meant is that the centres of these objects maintain a constant distance, though other points do not. Two lovers facing each other and chirruping lovingly may suddenly get annoyed and turn their backs to each other without going farther or nearer. Still their faces have turned farther and the backs nearer. Motion around a point also thus involves change of distance.

Further light is thrown on the relativity of motion by considering the following example. Suppose a dancing girl is moving around herself and a spectator is watching her intently. If we consider four of her positions, (1) she is facing the spectator to start with, (2) there is a time when the spectator is at her left, (3) there is a time when he is right at her back and (4) there is a time when he is at her right. The distance between them is always constant.

Now all these positions and the constancy of the distance can be explained by saying that the spectator is moving round the dancer and she is in fact stationary. If he describes a circle round her in such a way that he always faces her, and starts moving towards her left, all the above conditions will be fulfilled. If there is no stationary frame of reference of the earth, it will be impossible to say that the second description is wrong.

But we can still say that the second description is not so simple as the first. In the second description, we have to postulate two movements, (1) around the dancer and (2) around oneself; because in always facing the dancer the spectator has to move around oneself once. In the first description, the spectator keeps facing the dancer, simply by standing as he was. But in the second, he is first made to move in such a way that he cannot keep facing the dancer unless he cancels his change of position by another movement. This sounds like saying that

a man has a million minus a million rupees instead of saying that he has none.

It is clear from the discussion so far that unless there is an all inclusive space, and not merely a space defined by interrelations of objects, the relativity of motion cannot be avoided. This relativity of motion gives rise to relativity of time also, in the sense that without a stationary frame of reference, it is not possible to decide which of the two events is earlier. To illustrate this, I will simplify the example given by Russell. Suppose two brigands *A* and *B* stand on a railway line at a distance equal to the length of the train. The train is moving from *A* to *B*. At the closest range possible, *B* shoots the engine driver and *A* shoots the guard. A passenger in the central compartment hears *B's* shot first (because he is moving towards *B*) whereas a man on the railway line, standing in between the brigands at the centre hears the shots simultaneously. In such cases, we accept the version of the observer, who is stationary in relation to the earth, because the earth provides the stationary frame of reference in which all the other objects under discussion are included. But if such a frame of reference were not available, it will have to be admitted that events which are simultaneous for one observer may be successive for another observer[48.5] (this is a modified version of the example on p. 34-35).

This should not be interpreted to mean that there is no absolute distinction between past and future. In Russell's words again "Suppose an event *E* occurs to me, and simultaneously a flash of light goes out from me in all directions. Anything that happens to anybody after the light from the flash has reached it, is definitely after the event *E* in any system of reckoning time. Any event anywhere which I could have seen before the event *E* occurred to me is definitely before the event *E* in any system of reckoning time. But any event which happened in the intervening time is not definitely either before or after the event *E*"[48.5] (p. 43).

In other words even if there is a class of events whose temporal order is uncertain, there may be events which are earlier or later than all the events in this class. Thus with regard to a given event, some events will definitely belong to the future and some to the past.

Some people, misled by the the word observer, may argue that the observation of an event is not the event and it is not legitimate to infer that an event observed earlier, occurs earlier. In order to avoid this difficulty, we must replace the word observation by the word effect. Observation of an event is one of the many effects of the event. What can be said about observation can equally well be said about an effect of the event on a photographic plate or a sound recorder. Now, if we describe all the effects of an event, we can be legitimately said to have described the whole event. Thus, if in one frame of reference all the effects of an event E_1 are later than some effect or effects of an event E_2, it is reasonable to suppose that in that frame of reference E_1 itself is later than E_2.

Just as the relativity of motion gives rise to the relativity of time, it gives rise to the ralativity of shape also. The motion of a body can be represented by a path. This path can be straight or curved, etc. Now just as a body moving in relation to one frame of reference may be stationary in another frame of reference, a body moving in a straight line in one frame of reference may be moving curvilinearly in another frame of reference. The following example makes this clear.

Suppose a lift is freely falling in a well. The walls of the lift are in contact with the walls of the well. On one wall of the lift there is a narrow horizontal slit. A man in the lift moves a chalk along the slit. Now the movement of the chalk will draw a straight horizontal line on the wall of the lift, but a parabola on the wall of the well. The same identical movement thus traces paths of quite different shapes in different frames of reference.

Upshot :

We may now take stock of what has been said so far.

It has been shown that there is no formidable logical objection against defining motion in terms of change of place. It can alternatively be defined as change of distance between two objects. This latter definition brings in the relativity of motion. The arguments which go to show that motion is not merely relative have been found to be inadequate though a meaning can be assigned to the motion of A as such, if A is spatially included in a class of objects. The relativity of motion gives rise to the relativity of Time and Shape, since like motion, these also depend on a frame of reference.

IV

Shape and Size

It is by now clear that both shape and motion are relative. But the relativity of shape is slightly different from the relativity of motion. The motion of A always implies a B in relation to which A moves, but the shape of A does not necessarily imply a B in relation to which it has a given shape. A itself is a complex of terms. The relations of these terms determine the shape of A. The shape is relative because these relations themselves may depend on terms outside A. The successive positions of the chalk is the A in question. The shape of this A depends on the relations of these positions with each other. But these relations themselves are curvilinear when the positions are taken in relation to the lift. O and E may be husband and wife when they are considered in relation to their home, but they may be — in very fortunate cases — colleagues if considered in relation to their place of work. Thus the relation between two terms may change with the frame of reference.

But it is not a logical necessity that the relations of the terms of A must change with the frame of reference. The shape of A will change with the frame of reference only if these relations change with the frame of reference. Otherwise it will be a constant property of A.

The crucial difference between the shape of A and the motion of A, thus lies in the fact that the proposition "A moves" is not meaningful unless we specify the B in relation to which it moves, but the proposition "A is straight, or curved" is meaningful even if we do not specify any terms outside A.

In spite of this, shape still remains "relative" in the sense that it is a property of the relations obtaining between the terms of A.

The Properties of a Straight Line :

Though one is accustomed to use the word shape and the adjectives straight, curved, etc., only in connection with space, the notions signified by these words are not peculiar to space. They are quite general and applicable in other realms. Let us take the notion "straight" for example. This notion is complex and implies *at least* four different conditions. A straight line is supposed to fulfil all these conditions. The conditions are—

(1) If we take any three points on a straight line, we can form three pairs of points out of them, such that the relations of order between the points within all pairs are identical. Thus if the line is running from east to west, among the points ABC on it, B is to the west of A, C is to the west of B and C is to the west of A. There being an identical relation of order within all pairs formed out of these points, any two points completely define a straight line. This property can be called unidirectionality.

(2) If we take any two points A and B on a straight line, there is a distance between them. If we join the points by any other line, the distance between the points along this line will be greater than the distance along the straight line. This property can be called shortestness.

(3) Two points on a straight line are directly related, and do not require any third term to define this relation. Being "to the west of a" is a direct relation between A and B, we

require no third point to define this relation. But "being to the north-west of" would require a third point *viz.*- a point to the north. The relation "north-west of" is therefore indirect. The property of not requiring a third term can be called directness.

This property should not be confused with shortestness. A relation can be said to be shorter than another only if the former forms a part of the latter. But a direct relation may not always form part of an indirect one. In such a case, it cannot be said that the indirect relation must be longer simply because it is indirect.

(4) The first point on the straight line is not also the the last point. This happens on a circle. The circle begins and ends at the same point. The property which excludes this can be called *openness*.

These four properties are not peculiar to a straight line in space. For example (i) unidirectionality is present in the 'line' of kings of a country. If we take any three kings in the line, they are governed by the asymmetrical transitive relation "successor of", such that *B* is a successor of *A*, *C* is a successor of *B*, and *C* is a successor of *A*. Here the relation between any two kings is identical. The line of kings always maintains the same direction and therefore it can be called a straight line, though non-spatial.

(ii) The property of shortestness requires that a relation should form part of some other relation between the given terms. This property is not a peculiarity of points on a straight line. The five brothers who married Draupadi* exemplify how even family relations possess this property. The younger brother of a husband is called *devar*, and the older is called *shwashura*. Now Draupadi had two relations to her eldest husband *i. e.* he was a husband as well as a *shwashura*, and two to her youngest *i. e.* he was both a husband and a *devar*. But the remaining three had three relations, they were (1) husbands

*The heroine of the Mahabharat.

(2) *devars*, as well as (3) *shwashuras*.* Now of all these relations of Draupadi within this family of six, that of husband obviously forms a part of all the others — because, all the other relations have to be defined in terms of this relation. This relation therefore is the shortest (or nearest). Shortestness thus is a property possessed by other relations besides distance in space.

(iii) "*A* is an enemy of *B*" states a direct relation between *A* and *B*, because we do not have to mention any terms besides *A* and *B* to define the relation. But "*A* is the brother-in-law of *B*" states an indirect relation, because the relation "brother-in-law" cannot be defined without mentioning the wife, a third term besides *A* and *B*. Still I suppose nobody would suggest that "enemy" is a "shorter" relation than brother-in law. Where one relation is not part of another, there is no way of deciding which is shorter and which is longer. Thus the shortest distance between two points is only one of the cases of the shortness of a relation, and non-spatial relations can be easily shown to possess shortness.

(iv) The property of openness is possessed by an individual's life composed of three stages : childhood, youth and old age. The first stage here is not also the last. We cannot get back to our childhood by becoming older. Openness again is not thus a peculiarity of a straight line in space.

The four properties are not only not a peculiarity of a straight line in space, they are not even inseparable. This can be illustrated in the following way :—

I. Suppose there is a tea stall in which the customers have to move in a queue to get their full cup of tea. You pick up your cup and saucer at the first counter, turn the tap at the second counter to get your decoction, get your milk similarly from the third, and your sugar from the fourth and finally pick

*पतिश्वशुरता ज्येष्ठे, पतिदेवरताऽनुजे ।
इतरेषु च पाञ्चाल्यास्त्रितयं त्रितयं त्रिषु ।।

up your spoon from the fifth. This makes one cup of tea. But you are very much fond of tea and want to drink at least four cups. According to the rules of the stall, you have no alternative, but to repeat the process four times. In your four rounds, the cup, decoction, milk, sugar and the spoon are arranged in such a way that the arrangement has all the properties listed above excepting that of openness. After taking your spoon you come back to the cup again.* In an "open" or noncyclical arrangement, you cannot come back to the term you started from.

The arrangement has the property of unidirectionality, because out of any two terms one succeeds the other. There is an identical order relation within all pairs of terms.

Suppose the path of the queue is so narrow that two persons cannot stand side by side, so that you can neither skip up the positions in the queue nor go back. Once you enter you must go the entire length of the queue. Under such conditions, the arrangement will have the property of shortestness also, because taking any two terms, say the decoction and the sugar, the shortest way from the first to the second lies along the queue. Any other way will be longer, because you cannot, avoid going along the queue by following any other way.

There is a direct relation between any two terms, say, the cup and the milk. We can say that the milk succeeds the cup without mentioning any third term.

II. Suppose in a ceremony the wives of military officers are seated according to the seniority of their husbands. This seating arrangement will have all the properties excepting directness. The arrangement is not based on any direct relation between the wives. Their positions in the arrangement cannot be explained without mentioning the seniority of their husbands.

*All cups, decoctions etc. are to be regarded as identical.

This arrangement has the property of unidirectionality, because if we take any two wives, one of them is the wife of a senior officer, (assuming that officers of equal seniority are not present). There is an identical relation between any two. It has the property of shortestness, because if we arrange the wives according to the seniority of the brothers-in-law of their brothers, (who are supposed to have only one brother-in-law) we get the same arrangement, but our basic relation forms part of this new relation, and is therefore shorter. It has obviously the property of openness, because the arrangement ends with the wife of the junior-most officer and there is no going back to the wife of the senior-most officer from there.

III. Suppose we square the ages of school boys and then arrange them putting first the boy with the largest square of age, second the second largest square of age and so on, the basic relation of our arrangement will lack shortestness, because the difference in age will give the same arrangement as difference in the squares of ages and the former difference forms part of the latter. But apart from shortestness the arrangement has the other three properties.

It has the property of unidirectionality because out of any two boys one is the older. It has the property of directness, because the relation "order" can be mentioned without mentioning any third term. It has the property of openness, because there is no going back from the youngest to the oldest boy.

IV. Suppose we first take A, then take A's father, then take A's son, then A's father's father, then A's son's son and so on. The plan of our arrangement does not have unidirectionality. The relations between terms within all pairs are not identical. They are constantly changing.

If instead of A's father, we take his uncle's* brother and in place of A's son, his brother's* nephew, we shall get the same arrangement, but this new relation will include our basic relation as a part, since 'uncle' cannot be defined without 'father' and 'nephew' cannot be defined without 'son'. This basic relation is thus shorter.

There is a direct relation between any two terms because out of any two one is either an ancestor or a descendant of the other. Other terms are required to state the exact relationship, but not merely to state ancestorhood or descendanthood. The arrangement is 'open', because there is no coming back to the first term.

It will be noticed that in the case of shortestness we have always mentioned a relation as compared to which the given relation is shorter. The relation of distance between two points on a straight line is not shorter than all possible relations between these points. It is only shorter than other 'distances' between those very points, that is, " shortestness " is relevant when we are comparing relations of the same type, which are such that one can be said to be a part of the other. A given relation between two terms can be said to be not the shortest, only if a relation which forms part of it holds between the same terms.

We can thus deliberately choose relations which lack one of the properties of straight relations and form an arrangement which has the other three properties of a straight line. The properties can therefore be regarded as independent.

The notion of 'straight' has been analysed here in order to illustrate how shape is a property of relations. That shape is 'relative' merely means that the same terms can have one set of relations in one frame of reference, and another, in another. Those who are partners in one game may be rivals in another.

*The uncle is to be assumed to have only one brother and the brother is to be assumed to have only one nephew.

Equality of Size :

Just as motions can be compared with regard to shape, they can be compared with regard to quantity also. One motion may be longer than another, or faster than another. In order to be able to say that A is faster than B it is necessary that A requires less time to cover a distance and if A is said to move with the same speed throughout a period, it must be possible to show that it covers equal distances in equal times. Bertrand Russell has raised a question here, as to how equality of distances and the consequent uniformity of motion are to be defined[48·4] (p. 482). This question is logically and philosophically important.

In practice, we place a yard on a piece of cloth and say that the portion covered by the yard is a yard long. Thus the quality of the yard and the cloth is established by overlap. The logical idea involved in this overlap is hard to analyse, but the following seems to be an attempt in the right direction.

Spatial overlap is analogous to simultaneity in Time. Time may be regarded as a single line of events. Now two simultaneous events, though not identical as events occur at an identical time. Similarly we may take a length in space and ignore all other dimensions. In such a space overlapping lengths can be regarded as occupying the same space though they are lengths of two different objects. In practice, the yard and the cloth occupy different places, but the difference is with regard to a second dimension, which has to be ignored in order to conceive overlap or congruence. We may also use the word juxtaposition with regard to the end A of the cloth on which is placed the end X of the yard. Let the cloth and the yard on it be placed from left to right and let us ignore all other dimensions besides length. Now out of X and A neither is to the right or left of the other, and A has the same relation to X as X has to itself in the class of points comprising XY, i. e. the length of the yard.

Once the overlap of X and A is conceived, the other points present no difficulty. They must at least fulfil the following conditions :—

(1) Every point to the right of X is to the right of A and *vice versa*.

(2) Every point to the left of Y is to the left of B and *vice versa*.

(3) There is no point to the left of X, or A, and to the right of Y or B.

(4) For every point in XY there is a point in AB which is juxtaposed to it and *vice versa*.

(5) With regard to juxtaposed points p_1 and p_2 between the end points X, Y and A, B respectively, a point to the left of p_1 is to the left of p_2 and a point to the right of p_1 is to the right of p_2 and *vice versa*.

(6) XY and AB have no other points.

These six conditions can be present where juxtaposition is lacking as can be seen in the case of a stretched rubber belt. If the original belt is designated as AB and the stretched one as XY, all the above conditions hold because the points of AB and XY are identical. But a belt overlapping the original belt cannot overlap the stretched belt. Like simultaneity overlap is a transitive relation. If a overlaps b, but not c, it is to be supposed that b does not overlap c.

Another point to note, in connection with the six conditions is that the relation between X and Y, A and Y has got to be identical, or more generally a point juxtaposed to any point P has the same relation to Q, as P has to Q.

The reader might think that this is impossible because the yard is placed on the cloth and therefore B, the end point of the cloth, does not have the same relation to X, the first point of the yard, as to A the first point of the cloth itself. B may be to the left of X on a *lower plane*, whereas it is to the left of A *in the same plane*. This difficulty arises, as will be explained

more fully while dealing with points and instants, because the abstraction necessary for considering the relations of points has not been made. Since it is physically impossible for the yard and the cloth to occupy the same place at the same time, the yard has got to be placed on the cloth, but this difference in the places of the cloth and the yard is not *logically relevant* to the properties of the relations defining congruence. Even when the difference in the places is reduced indefinitely, the relations between the points in XY, to each other, and the points in AB to each other do not change. It is only these relations that are necessary for the definition of congruence, and the relations arising out of the *physical impossibility* of placing the yard and the cloth in the same place are irrelevant.

The relations arising from keeping the yard on the cloth arise in addition to, and not in place of the relations necessary for congruence. If the yard and cloth were exactly in the same place, B would be to the left of X but not below. By placing the yard on the cloth, B is not one whit less to the left of X, but in addition, it is also below X. B belongs to two classes of points :

(1) those that are to the left of X.

and (2) those that are below X. Thus the relation necessary for congruence, *viz.*- 'to the left of ' is preserved intact. If additional relations are also present they do not detract from the congruence.

That the relation 'below to the left' is not a new relation compounded from 'below and left' will be clear if we compare it with the relation 'maternal grandfather'. The maternal grandfather is the father of the mother. This is a new relation, because the grandfather is neither the mother nor the father. Unlike the relation ' below to the left', 'grandfather' defines a class distinct from mothers and fathers. The relation is not in addition to but in lieu of the relations 'mother of' and 'father of'.

This irrelevance of the additional relations arising out of difference of place will be clearly illustrated if we take a non-spatial example of congruence. There is perfect congruence between the hierarchies of Air-force and Army Officers (ignoring their order of precedence).

(1) The highest Army and Air Officer are juxtaposed, *i. e.* neither of them is above or below the other, and they have the same relation to each other, *i. e.* of equality, which they have to themselves in their respective services.

(2) Every officer below the General is below the Air Chief Marshal and *vice versa.*

(3) Every officer above the Second Lieutenant is above the Pilot Officer and *vice versa.*

(4) There is no officer above the General or the Air Marshal and below the Second Lieutenant or the Pilot Officer.

(5) For every officer-rank in the Army, there is a juxtaposed, *i. e.* a corresponding equal rank in the Air Force and *vice versa.*

(6) With regard to juxtaposed, *i. e.* equal ranks, R_1 and R_2 a rank below R_1 is below R_2 and rank above R_1 is above R_2 and *vice versa.*

(7) Army and Air Force have no other officer ranks.

This example illustrates how 'places' are irrelevant for congruence relations. Air Force and Army are analogous to the different places of the yard and the cloth. But still the relation of a Wing-Commander to a Squadron Leader is the same, as to a Major and *vice versa.*

The measurement of the cloth by placing the yard on successive portions of the cloth, involves two further assumptions, *viz.*-

(1) all portions congruent with the yard are congruent with one another; and

(2) a yard in one place is congruent with the same yard in another place, (if there is no qualitative difference in the two places).

The second assumption will be untrue only if rods which were congruent in one place cease to be congruent in a new place. It may be suggested that the second assumption can be untrue even if all the rods expand or contract equally. But in such a case there will be no way of discovering the expansion or contraction. One may as well advance a theory that motion or no motion everything is always expanding and contracting, but since expansion and contraction occur without affecting the relative size of the objects, it is impossible to detect its presence!

In order to conceive such an absolute expansion, whose presence cannot be detected, we need an all inclusive space, just as in the case of absolute motion. The statement that A and B move in an absolute space, such that the distance between them increases, has a definite meaning *viz.*- that the initial distance between A and B is a part of their subsequent distance. On the other hand, if there is no all inclusive space, but only objects having spatial relations, this meaning cannot be assigned to the increase of distance between A and B. If A and B are the end points of a rod, an increase in the distance AB is what is called the expansion of the rod. Now if the space occupied by AB is nothing but the extension of AB itself, there is no question of anything forming part of anything else. There is only one distance, the extension of AB, and we cannot talk of an expansion or contraction of AB, unless we have some other object which was at first congruent with AB and is now only a part of it. The statement that "all objects have expanded equally" has no meaning on the relational theory of space.

This suggests that congruence can also be defined by the relation of inclusion. A is congruent with B means that A is included in B, and B is included in A, *i. e.* neither is a proper

part of the other. When two triangles are congruent, they can be placed on each other in such a way that, a side of the one is also a side of the other. It should be specially noted that the sides are identical and not merely similar. As will be explained more fully later, a yard cannot be identical with the portion of the cloth on which it is placed, but one geometrical line can be identical with another in the sense that the same line may belong to two different figures. The diagonal of a square is a line common to two triangles.

Taking a nonspatial example, we may say that the classes of—

(1) featherless bipeds; and

(2) rational animals, are congruent, because a member of the one class is also a member of the other and *vice versa*. Here the relations between the terms are not relevant as the classes are taken merely in their extension.

After defining equality of distance by 'congruence', we can define 'uniform' motion as that which covers equal distances in equal times. Uniform motion has one of the properties of a straight line, *viz.*- that two instants are sufficient to make the whole motion determinate. We can determine the position (*i. e.* its distance from its other positions) of an object in uniform motion at any time, by knowing its position at two other times. These two times may be prior or subsequent or one prior and the other subsequent to the position that is to be determined. Russell thinks, this contradicts the law of causation because causality is a relation between two terms, of which the one that determines, precedes and the one that is determined succeeds. Instead of this we have here three terms and the two terms that determine do not definitely precede what they determine[48.4] (p. 486).

I think the distinction between *Karak Hetu* and *Jnyapak Hetu* drawn by Indian Logicians is relevant here. Conditions

which bring *A* into existence are the *Karak Hetu*, whereas conditions which enable us to infer the existence of *A* are the *Jnyapak Hetu* of *A*. *Karak Hetu* must precede the effect but a *Jnyapak Hetu* need not. Again all the conditions necessary for the creation of *A* are together called the cause of *A*. Thus though the cause as cause is one it may consist of many factors. It is therefore difficult to see how causality is contradicted by the laws of motion.

Upshot :

It has been shown that shape arises out of particular types of relations and is not necessarily a property of Space. The notion of straight has been analysed to show that it consists of at least four independent conditions. The equality of size is a more difficult notion, but the lines on which it can be analysed have been indicated.

V

The Fundamentals of Measurement and their Applicability to Psychology

In considering size we naturally come to measurement. The word size is generally used in connection with space and therefore it is supposed that measurement is not applicable outside the realm of space and that "the talk of mental measurement can at best be figuratively meaningful". It is, therefore, necessary to analyse the fundamental concepts of measurement and to see whether there are any inherent difficulties in making it applicable to the mind.

Any property which can be expressed by the comparative degree, such as taller, heavier, more intelligent, more beautiful, etc. is measurable. *N* objects can be said to have been measured with regard to the property *P*, when they are arranged by an asymmetrical, transitive and connected relation expressed by the comparative degree of that property.

Members of this order differ qualitatively from each other, unlike the members in some other orders such as the spatial, the order of points from east to west for example. Every point in this order is qualitatively identical with every other point. Beyond the fact that out of any two points, one is to the west of the other, there is no other difference in the points.

It is clear that measurement in the above sense is applicable to the mind, because we can arrange a number of persons in an order with regard to their intelligence or musical aptitude, etc.

Degrees of Measurement :

Measurement of a higher degree requires not only that the objects should be arranged in the order but also that differences between the objects with regard to the property should also be arranged in an order. In other words, not only that we should be able to say that *A* is more intelligent than *B* and *B* is more intelligent than *C* but also that the difference in the intelligence of *A* and *B* is equal to, more or less than the difference between the intelligence of *B* and *C*.

The difference between *A* and *B* is independent of the number of terms between *A* and *B*. Even if there are no terms between *A* and *B*, and *B* and *C*, the difference *A*—*B* can be more or less than the difference *B*—C. Even if there is no person less intelligent than *A* and more intelligent than *B* and less intelligent than *B* and more intelligent than *C*, the superiority of *A* to *B* can still be more or less than the superiority of *B to C*.

Measurement of a still higher degree requires not only that equalities and inequalities should be established between differences but also between the property of an object and the combination of the same property of two or more other objects. In other words, we must be able to say not only that *A* is more intelligent than *B* and *C*, but also whether his intelligence is equal or unequal to the combined intelligence of *B* and *C*. If equality between *A* and a combination of terms smaller than *A* — say a — can be established, we can say not only that *A* is greater than *a* but also that it is *n* times greater than *a*.

The combination that can construct the bigger out of the smaller is of a specific type. Two one-inch chains can make

one two-inch chain if the end of the one is fixed to the end of the other, not if they are placed side by side. The usual argument that age is not additive because two 8 year-old girls do not make a damsel of 16 is therefore not correct. Addition of ages is not effected by considering the age of two persons. But if we have evidence from one source that *A* lived from 1900 for 25 years and from another source that he lived from 1925 for 25 years, we can logically infer that he lived $25 + 25 = 50$ years. Age is additive in this sense. Similarly, if we know from one source that a person has solved 50 problems in a test and from another source that he has solved another 50 problems of the same test, we can infer that his score on the test is $50 + 50 = 100$.

Intelligence is additive even in a more direct sense. Suppose *A* can solve problem 1 but not 2 and *B* can solve 2 but not 1 while *C* can solve both. It can then be said that the intellectual performance of *A* and *B* is together equal to that of *C*.

Where the variable itself is not additive what is correlated to it may be additive. For example, the number of birth-day-cakes hitherto used by one 16 year old may be equal to the number of birth-day-cakes used by two 8 year olds.

Additivity implies ratio judgements, *i. e.* judgements as to how many times one is of the other.

Ratio judgements imply a zero. If *A* is twice *B*, it follows that if *B* is taken twice from *A* nothing *i. e.* zero remains.

But this zero need only be an origin from which measurement starts and not the zero which is defined as the class whose only member is the null class *i. e.* entities such as the satellite of the moon or the son of a barren woman. The zero of temperature for example is not complete absence of temperature but the temperature at which water freezes. The temperature of all other things is measured in terms of difference from the freezing point. The freezing point is thus an origin from which all

temperatures are measured. Now when we say 50° is twice 25° what we mean is that if 25° are taken twice out of 50° the difference from the freezing point is zero. Similarly, we can say that a man who died at 50 lived twice as long as the man who died at 25. It means that if we take 25 years twice from 50 years, the difference in years from the birthday of the man who died at 50 becomes zero.

The notion that absolute zero is essential for ratio judgements arises from the following instance.

Suppose we regard shepherd S_h who has 50 sheep as an origin for counting the number of sheep any shepherd has. Now S_i who has 100 will be said to have 50 and S_j who has 200 will be said to have 150. 200 is twice 100 and S_j really has twice as many sheep as S_i, but according to the new reckoning S_i has 50 and S_j has 150 and 150 is not twice 50.

This falsification of ratios, however, arises not from shifting the origin from absolute zero to 50, but, from forgetting that what is now being measured is not differences from the absolute zero but from the number possessed by S_h. The ratio that is true of the difference of the number possessed by S_j and S_i from absolute zero need not be true of their difference from S_h.

We may therefore say that placing the origin at absolute zero is only a special case of measurement from an origin.

This does not imply that ratio judgements cannot be made if differences are taken not from absolute zero but from some other number. We can very well say that the difference S_j—S_h is 3 times the difference S_i—S_h.

Ratio judgements can be made without an absolute zero. On the other hand, ratio-judgements can be falsified even if absolute zero is retained. Suppose we count the sheep in terms of squares. Then one who has no sheep will still be said to have zero sheep since the square of zero is zero. So the absolute zero is retained even in the new system of counting but one who has

2 sheep will in this new system be said to have 4, and one who has 4 will be said to have 16. Now 4 is twice 2 but 16 is not twice but four times 4. Retention of the zero has not thus helped in preserving the ratios.

Zero is thus neither a ncessary nor a sufficient requirement for ratio-judgements. What is essential for such judgements is that the bigger can be constructed by a combination of the smaller.

A still higher degree of measurement requires that there should be no exception to this rule, *i. e.* there should be nothing so small that it cannot be constructed out of something smaller and nothing so big that it cannot enter as a constituent of some thing bigger. In other words, the property to be measured should be continuous.

It should not however be supposed that continuity always implies additivity. For continuity it is enough that there should be no smallest and no greatest and if *a* is greater than *b*, there should be something greater than *b* and smaller than *a*.

If intelligence is measured by the number of problems solved, it cannot be regarded as continuous because there could be no degree of intelligence, higher than that required for solving say, ten problems but lower than that required for solving eleven problems.

If intelligence is to be regarded continuous, problem-solving cannot be supposed to be an all or none function. It has to be supposed that among those who fail in reaching a solution, some are nearer the mark than others. Even those who have solved the problem can be differentiated from each other by the time taken for the solution.

Equality :

Additivity implies equality. If *a* can be constructed by combining *b* and *c*, it implies that *a* is equal to *b* plus *c*. When we measure length, we establish this type of equality between

the measured and the number of units in a scale. Equality must therefore be defined.

Equality is of various kinds :

(1) First is numerical equality. Two classes *x* and *y* can be said to have an equal number of members, if there is a one-one ralation between the members of the two classes. For example, if in a meeting every chair is occupied and no person is without a chair, the number of chairs is equal to the number of persons.

(2) The second is causal equality. The weight of 10 pounds of sugar and 10 pounds of salt is said to be equal, because when any one of them is hung on the weight-measure it brings the pointer to the same place.

(3) The third is equality by congruence. A one-foot-line is said to be equal to a foot-rule, because the foot rule and the line overlap each other. This overlap is not identical with physical contact. It can be defined thus.

A class *a* of terms in order is equal to another such class *b* when for every term in *a* there is a corresponding juxtaposed term in *b* and vice versa (see page 19).

A term of *a*, say a_t is said to be juxtaposed to a term of *b*, say b_t when a_t does not have the order producing relation to b_t, but has the same relation to all the other terms of *b* as b_t.

Suppose *a* is a class of points in an east-west line, and *b* is a line parallel to it. If the first point of *a* is neither to the east nor to the west of the first point of *b* it is juxtaposed to it. If this is true of the last points of *a* and *b* as well, the last points are also juxtaposed and if every other point on *a* has a corresponding juxtaposed point on *b* and vice versa, the two classes of points, *i. e.* lines can be said to be equal.

It may be supposed that if the first point on *a* is neither to the east nor to the west of the first point on *b*, it may be to the north or to the south and therefore a point which is to the west of the first point of *a* is not to the west but to the north-west or south-west of it. The subsequent points on *a* therefore cannot have the same relation to the first point of *b* as they have to the first point of *a*.

This is an error. What is north-west or south-west does not cease to be west. If a man has married his cousin she is his cousin-wife; but the cousin-wife is as much a wife as any other wife. Being a cousin is an additional relation and it is not relevant where we are only considering wife-hood. Similarly, being to the north or to the south is a relation additional to being to the west. When it is said that *A* and *B* have an identical relation with *C*, it does not follow that all the relations of *A*, *B* and *C* are identical. It is only the relation to the west of the first points of *a* and *b*, that is supposed to be identical, not any other relation.

Overlap is essential for *ascertaining* equality, but not for equality itself. Two rods can be equal even when they are not overlapping.

The rods themselves can be arranged by the order-producing relation. If the relation concerned is "to the west of", one rod is placed to the west of the other, and the last point (or rather surface) of the first is "overlapping" the first point (surface) of the second. The rod *b* which is to the west of the rod *a* has only those relations to *a* which the western points of *a* have to its own eastern points, i. e. *b* is only to the west of *a* without being to the north or south, etc. When these conditions are fulfilled, the two rods together can be said to be equal to one rod twice longer than either of them. Measurement of length thus consists in establishing equality between a length and a number of units suitably arranged.

Equality of intelligence is causal equality. Two persons are said to be equal in intellegence when they can solve the

same problems. Mere equality in the number of problems solved would not establish equality of intelligence, if the problems are different or not parallel (*i. e.* of equal variance validity, etc).

Measurement of a property which is manifested in various ways presents some peculiar problems. Take for example temperature. This is manifested to the sense of touch, in the change of state from solid to liquid and to gas, and in the phenomenon of expansion, etc. All these are said to be manifestations of temperature because the estimates based on these manifestations are correlated. We may arrange things in order of temperature by touch, or by relative solidity, or by the expansion caused. These three arrangements have more or less the same order.

But when the order is not the same, we very often regard one as correct and the other as wrong. The reason is the following.

If we classify objects which are either near the boiling or the freezing point as hot and cold by the sense of touch and by the thermometer there is perfect agreement between the two classifications. But when the difference is small according to the thermometer, we find that the classification based on the sense of touch does not agree with the one based on the thermometer. In this case, we regard the sense of touch as wrong, because persons whose senses of touch agree perfectly in detecting large thermometer-reading-differences begin to disagree where small thermometer-reading-differences are concerned. In other words, if *A* is hotter by a degree than *B* according to the thermometer, one person may say that *A* is hotter, another that *B* is hotter and the third that they are equally hot. The sense of touch therefore can be regarded as unreliable at this level.

In the case of small thermometer-reading-differences, it may also happen that none of the three persons may notice any difference and agree that the temperatures are equal. In

this case, unreliability cannot be ascribed to the sense of touch. Why then should we suppose that the differences revealed by the thermometer are "real" and not artifacts produced by the thermometer, and that the equality appearing to the sense of touch is due to its obtuseness ?

Here the test of truth is the same that we use to distinguish perception from hallucination. The object revealed in hallucination does not have any effects not mediated by hallucination. If a snake is seen only in a dream, the snake has effect only on the dreamer and can affect anyone else only through the dreamer. But if it is seen in perception, it can affect even those who have not seen it without the instrumentality of the person who has seen it. A perceived snake can bite those who have not perceived it. Effects unmediated by the perception are thus the mark of perception as distinguished from hallucination.

Now the point at issue is whether the difference revealed by the thermometer is a hallucination. If it is a hallucination two things which have different temperatures according to the thermometer will not have different effects excepting on the thermometer or through the thermometer. But it can be shown that they have direct effects even on the sense of touch. The thermometer may reveal a difference in the temperature of solid and liquid oil, though to the touch the temperaure appears the same. But the touch can judge that one is solid and the other is liquid. In other words, things which are at different temperatures according to the thermometer, affect the sense of touch differently even when the difference in temperature is not noticed by touch. Thus the touch contradicts itself in not noticing the difference in temperature but noticing the *effects* of different temperature, and the thermometer is therefore regarded as correct even at this level.

Purity of Variables :

One of the conditions of the validity of measurement is that the variable to be measured must be pure. If we are

counting the number of grapes, it is necessary to ensure that some berries have not been counted as grapes. If we are measuring intelligence, it is necessary that some purely information-items have not been included as intelligence-problems.

There is another type of purity which is difficult to obtain in the case of psychological variables and that is identity of ability at different levels. In order to say that a man who can solve 40 problems is twice more intelligent than a man who can solve 20, the problems must be parallel, *i. e.* they must be similar in content, the number of persons able to solve them must be the same, they must have the same correlations with other problems, and so on. Even then *A* who can solve 50 problems may not be able to solve some problems which *B*, who is able to solve only 40 — can solve. This situation is analogous to *A* and *B* commonly owning 30 houses, *A* owning 20 more, which *B* does not own, and *B* owning 10 more which *A* does not own. If the prices of all the houses are equal, *A* can be said to be wealthier in housing property than *B* though he does not own some of the houses which *B* owns.

A man solving a more difficult problem is not necessarily more intelligent. The difficulty of a problem is analogous to the rareness of a possession. A man may be possessing a very rare book, but he cannot be said to be richer in money than a person who possesses a less rare book which fetches a higher price in the market.

Physics has no privilege :

One of the objections to mental measurement is that a mental quality is not fixed, it fluctuates and therefore there is no point in measuring it. The answer is that the existence of fluctuation itself cannot be known without some kind of measurement, and it is worthwhile measuring fluctuation and finding out its maximum, minimum, central tendency, etc. It will not be worthwhile doing this only if the fluctuation is completely erratic in an individual and no inferences can be drawn

from it about the fluctuations in other individuals. This, fortunately, cannot be alleged against psychological variables.

Physical measurement appears to be more accurate than mental measurement but there are some considerations to note here also. Physical measurement is also unreliable for small magnitudes. If we have to measure weights which differ less than a small fraction, the correlation between repeated measures may be as low as between test and retest for intelligence.

It may be contended that though in certain ranges measurement of weight may be as unreliable as measurement of intelligence, if we take the entire range of intelligence and the entire range of weight, the measurement of weight will be found to be more reliable.

The contention may be sound but at present there is no way of demonstrating it.

Besides unreliability, another mark of error is grossness of approximation. In order to measure intelligence validly, by the number of problems solved, the problems must be parallel. In practice only approximately parallel problems can be found.

But this is so in the case of physical measurement also. In order to measure weight validly by the number of grams, the grams must be equal. So must be the units of length for measuring length. But these can be equal only at equal temperatures and with many other factors assumed to be constant. In practice, these equalities are only approximate.

There is another type of approximation which is inherent *viz.*- that involved in expressing the length of a curve in terms of a sum of the length of short straight lines or in expressing an irrational in terms of integers. Since a curve is NOT a sum of straight lines and an irrational is *not* a relation between two integers, such expressions can only be approximations.

Some readers are unnecessarily diffident about the application of measurement to intelligence by the fact that

differences in intelligence which appear equal by raw scores are not equal by percentile scores. The difference between a person who gets 60 and a person who gets 40 is equal to the difference between a person who gets 80 and a person who gets 60. But if we consider the percentage of the persons surpassed as an indication of the level of intelligence, this may not be so. Suppose the number of persons surpassed by getting a score of 80, 60 and 40 is p_1, p_2, and p_3, respectively. $p_2 - p_3$, may be greater than $p_1 - p_2$, though the score-difference is 20 in both cases.

But this difficulty is not peculiar to Psychology. It may appear in the case of height also if height in inches is converted into percentiles. The percentiles are only ordinal measures. They are not expected to reflect accurately the more direct measures like height in inches or intelligence-level in terms of score.

Sometimes it is contended that the same score difference does not stand for the same real difference at different levels. An increase from 10 to 11 feet in the length of a chain is not the same as an increase from one to two feet.

This contention confuses relation-measures with non-relation-measures. Two feet are twice one foot, but 11 feet are only 11/10th of 10 feet. In this sense, an increase of one foot over one foot is greater than the increase of one foot over ten feet.

But from this, it should not be inferred that the one foot over ten feet is somehow shorter than the 1 foot over one foot. The two 1 feet are equal since $2-1 = 11-10$. What is different is the bigness of 2 as compared to 1, and the bigness of 11 as compared to 10. The relation of 2 and 1 is here contrasted with the relation of 11 and 10, and not the difference of 2 and 1 with the difference of 11 and 10.

The discussion so far shows that there are no peculiarties of the the mind which make measurement less applicable to it than to physical objects. If mental measurement, today

is not as efficient as physical measurement, the reason surely is that humanity has devoted far less attention to Psychology than to Physics.

Upshot :

Measurement consists in arranging individuals having a certain property in a certain order. There are various degrees of measurement. All these degrees are meaningful with regard to Psychology even though in the present state of the Science, it may not be possible to use measurement of the highest degree in the case of many mental variables. There are no special theoretical difficulties, not encountered by Physics, in the way of mental measurement.

VI

The Relative and the Real

From the fact that motion and shape are relative, some people draw the conclusion that they are not real. The relationship of relativity and reality must therefore be investigated.

The notion that when the distance between *A* and *B* is changing, it *must* be possible to say who is moving and who is at rest, is so deeprooted, that many philosophers feel this to be bound up with the very reality of motion. Bergson remarks "We may not be able to say which parts of the whole are in motion", "motion there is in the whole none the less"[4.1] (p. 254). This remark cannot fail to remind one of the monistic doctrine of relations which holds that a relation is the property of the whole composed of the relation and its terms. In a sense a relation is contained in the whole, but it is the terms that are related and not the whole. Motion may be *in* the whole, but it is not the "whole" that moves. The proper answer to the question, "what is it that moves ? *A* or *B*, when the distance between them is changing?" is : "both *A* and *B* move". Motion is a symmetrical relation like equal. If *A* is equal to *B*, *B* is

also equal to *A*. It is absurd to suppose that the relation of equality is in the "whole" composed of *A* and *B*, and does not belong to the terms *A* and *B*. The reason why we cannot say that it is *A* that moves and not *B*, is that motion is a symmetrical relation, not involving this sort of exclusiveness. The symmetry of motion in no way involves its unreality.

Sometimes one meets with an argument on the following lines. Consider the proposition "Ram is tall". This proposition may be true, if the height of Ram is compared with that of the Pygmies, but may be false if it is compared with that of the Sikhs. It can be argued that a proposition which can be both true and false cannot be called true.

The mistake in this argument is quite obvious. The proposition as it stands is ambiguous and its meaning can be made definite only by specifying its context. When the meaning of the proposition is made definite, the proposition is as true or false as any other.

The reason why relativity creates a sense of unreality is that some relations are purely external. If *A* having a relation *R* is completely identical with *A* not having that relation, excepting for the fact that it does have that relation, the relation *R* is said to be external. In such a case even if we knew everything about *A*, except the fact that it has the relation *R*, we shall never be able to infer that it has that relation. If motion is an external relation, we shall never be able to infer that *A* is in motion even by knowing everything about *A*, as long as we do not know *B*, in relation to which *A* moves. Thus if the *As* with and without motion are completely identical, one is legitimately tempted to say that motion is not quite real as far as *A* is concerned. But this temptation must not be

allowed to mislead us into supposing that motion can be regarded unreal even if *A* and *B* are both taken into account, because *A* and *B*, in motion in relation to each other, are not identical with *A* and *B* at rest in relation to each other, even if motion is purely an external relation.

If motion is an internal relation, *A* moving in relation to *B* will have some properties, not possessed by *A* at rest in relation to *B*. In such a case, it will be possible to infer whether *A* is in motion or not from these distinguishing properties. But these properties will still not justify us in saying that it is *A* that moves and *not B*. Common sense has developed certain conventions about attributing motion and rest. Objects at rest in relation to the earth are regarded as absolutely at rest. But apart from such considerstions which have been discussed at length before, motion can be regarded as a symmetrical relation like brother of and not an asymmetrical one like wife of. The question : is *A* the brother of *B* or is *B* the brother of *A* is meaningless. But the question : is *A* the wife of *B* or is *B* the wife of *A*, is not only not meaningless, but can be answered definitely by knowing the sexes of *A* and *B*. Thus even if motion is regarded as an internal relation, as long as it is not regarded as an asymmetrical relation, the question whether it is *A* or *B* that moves cannot be experimentally settled.

This brings out an immensely important point. Even those who believed in absolute motion, thought of it, as occurring in space, or in other words the absolute movement of *A* was only the changing positions of *A* in static space. This sort of absoluteness does not quite get rid of the relativity of motion. If *A* changes its distance from a point *p* in space, the point

equally changes its distance from *A*. The fact that motion is a relation between *A* and *p* is not avoided even here. All that is asserted is that the relation is *not* symmetrical, *i. e.* the relation of *A* to *p* is not the same as that of *p* to *A*. But asymmetrical relations are still relations, and motion does not become less relative by bringing in static space. Those who suppose that the relativity of motion somehow robs it of its reality and therefore runs to refuge in absolute space should realize that there is no escape from relativity even there, and therefore had best reconcile themselves to views which regard relativity as not incompatible with reality.

The Reality of Relations :

A defence of relativeness cannot afford to ignore the fundamental objections of Bradley to relations themselves. His first objection is that relations cannot exist without the terms between which they hold, and at the same time they must be completely irrelevant to the terms. *A* and *B* which are related must be completely identical with *A* and *B* which are not so related. Otherwise if *A* and *B* change the moment the relation relates them, it will be something else that the relation has related and not *A* and *B*. Again a relation cannot change merely because its terms have changed. "A cat *in* the room" and "a horse *in* the stable" are different, but the relation of "inness" must be regarded as identical in both cases. All this is a mystery. A relation must have terms and yet the change of terms makes no difference to the relation[6] (p. 27).

If we take the bull by the horns and say that, relations are external to the terms though they cannot exist without the terms, it does not affect them in any way, we land ourselves

in an infinite regress. Even if the relation is external to the term, we have to say that the term *has* the relation. There is thus a relation between the relation and the term. "The links are united by a link, and this band of union is a link which also has two ends, and these require each a fresh link to connect them with the old"[6] (p. 28).

Bertrand Russell avoids this difficulty by saying that the infinite regress involved is not vicious, because it is not a part of the *meaning* of a proposition which asserts the relation. The proposition "*A* differs from *B*" may imply that "*A* has the relation of difference" and this may further imply that "*A* possesses the property of having the relation of difference" etc. But there is nothing to be afraid of in this. A proposition may quite legitimately imply an infinite member of other propositions. The vicious type of regress consists in requiring an infinite number of propositions for defining the very meaning of a proposition. The meaning of the proposition "*A* differs from *B*" does not require the infinite member of propositions listed above[48-4] (p. 348-349).

It is obvious that the meaning of a proposition is not the same thing as its implications.

(1) "*A* is greater than *B*" implies

(2) "*B* is less than A",

and vice versa, but the meaning of the two propositions is different. In many contexts, the one cannot be substituted for the other. The first and *not* the second is the correct reply to the question "What is the relation between *A* and *B*?"

It is possible to meet Bradley's argument by supposing that the relation of a relation to its term is identical with the relation itself, just as any power of 1 is equal to 1. If one

wishes to draw attention not only to that A is related, but also to that A has the relation and further to that A has the property of having the relation etc., one may go on using the relation of the relation of the relation etc., but this does not preclude the possibility that these are all identical.

Alternatively this may be expressed by saying that the infinite regress of Bradley is a regress of tautologies and therefore not vicious. One might ask "Is a table a table?" On getting a reply in the affirmative, one might further ask, "Is a table that is a table, a table?" and further "Is a table that is a table, that is a table, a table?" and so on..... Nobody would seriously suggest that this sort of regress lands the proposition "this is a table" in any difficulties.

Another relevant example would be the proposition that the proposition P is true. One might ask "Is the proposition, 'the proposition p is true' true?" and so on..... This is an infinite regress, but nobody is seriously perturbed by it.

Upshot :

By being relative, motion does not lose any of its reality. The arguments against the reality of relations can be easily answered.

VII

The Simple and the Real

It has been said very often that if A and B are moving in relation to each other, sometimes it is possible to get a simple description of facts by regarding one of them as stationary and the other as moving. Now if one description is simpler than another, does it entitle us to say that it is also truer than another ?

It can certainly be said that a simple description is less likely to be proven wrong, if by it we mean a description that asserts the minimum that is common to all descriptions. Taking the example of motion again take the following statements (1) A was in Delhi on Monday as well as on Friday, (2) A went to Bombay from Delhi just before Monday began and returned just before Friday began. The second statement asserts what the first asserts and in addition asserts that A travelled to Bombay and back. If the second is correct the first must be correct, but if the first is correct the second may not be, because A may not have moved away from Delhi at all.

But whenever one description is simpler than another, it may not always happen that it asserts what the other asserts as in the above case. The word 'simpler' may have the following meanings (1) the terms and relations involved in

the descriptions are more familiar, (2) it involves less number of terms and relations. When we describe the proportion of the literates to the illiterates in a country, by a percentage our description becomes more familiar and easier to understand. But the description in terms of percentages is not truer than the description in terms of raw figures. In fact, one who wants the whole truth may want the raw figures in addition to the percentages. Trying to describe everything in familiar terms may make the description distorted, because what is described may not be capable of being forced into familar terms.

A familiar description may involve a larger number of terms than an unfamiliar one. If *E* is described to *A* (1) as a sister of his wife's friend and as (2) a Tasmanian, the former description may be more familiar to him though it contains a larger number of terms and relations. Still the second description may be simpler in the sense of being less complex. (We here assume that Tasmanian, friend, etc., are simple terms and ignore their further analysis).

Are less complex descriptions in any sense truer than more complex ones?

It can be said that it is easier to falsify complex descriptions. On the other hand, it is easier to verify the simpler ones. Even if *one* of the terms and relations constituting the complex description is false, the description is false. It is therefore obviously easier to establish the truth of simpler descriptions, as they contain fewer terms and relations.

In a special case, it may be more difficult to falsify a complex description as compared to a simpler one. This happens when the elements of the description are related to each other by implication, and many of its terms are already known to be true. If I hear the horn of a car, I may describe the car as (1) having four wheels, with the front wheels capable of being steered etc., or as (2) black. This first description is more complex but is far more difficult to falsify than the

second. We may even say that the first is far less likely to be false than the second.

Sometimes the opposite of the simple is complicated and not merely complex. We may write the two series (1) 2, 4, 8..... and (2) 8, 9, 10....in the following two ways :—

I. 2 4 8 .. ..
   8 9 10 .. ..

II. 2 8 4 9 8 10

that is, we may place the first term of the second below the first term of the first etc., or we may place the first term of the second to the right of the first term of the first etc. It will be difficult to argue that the first arrangement contains fewer elements than the second and yet the second is in a sense more complicated, because it is more difficult to disentangle the two series in it. Thus some elements in a whole may have the property of masking other elements. When such elements are present in a description, the description can be said to be more complicated.

Are simpler descriptions truer than more complicated descriptions?

The answer to this question is obvious. A description that masks the elements that are present is like a partisan historian's description of his hero, which creates an air of innocence around the hero's objectionable deeds. Since it tries to hide some real elements, it cannot be said to be as true as a description which presents all the elements clearly.

Sometimes an intentional description is said to be simpler than an extensional one. We might describe the series 2, 4, 8,.....64 by (1) mentioning all the terms in it in the given order. This is extensional description (2) We might describe it by mentioning the first term, the number of terms, and the law of formation of the series. This is intentional description.

The intentional description is simpler, because it is easy to state and remember even if the number of terms in it is very large. It would be almost impossible to state and remember an extensional description of a series containing about 100 terms or more.

An intentional description is truer in the sense that it enables us to infer the remaining terms by knowing a few terms. It is thus a tool for discovering new truths. It is not possible to infer anything from an extensional description. Its truth is thus restricted to what is already known to be true. Science is therefore primarily interested in intentional descriptions.

Reverting to our example of the dancer and the spectator, if we were content with extensional descriptions we should only mention all the positions of the dancer and the spectator at different times and not talk of any motion, whether that of the dancer around herself or of the spectator around the dancer. But this will not enable us to predict their unknown positions from the known ones. The description therefore will not be scientifically useful. Descriptions in terms of the paths of motion thus become imperative.

One of the properties of intentional descriptions is that many intentional descriptions are applicable even where only one extensional description applies. In order to choose between two alternative intentional descriptions, we must extend the domain of the extensional description until one of the two ceases to be applicable. Let us take the following series :—

A	B	C	D	E	F	G	H	I	J	K	L
1	3	5	2	4	6	1	2	3	4	5	6

If we only know the terms upto C, we may describe the whole series as (1) the series of odd numbers (2) the first three terms of a series of odd numbers followed by a series of even numbers left out (3) the first three terms of a series

of odd numbers followed by a series of even numbers left out, followed by a series of all the integers included in the first two series. When we know the terms upto D, the first description is no longer applicable; and when we know the terms upto L, the second likewise ceases to be applicable. The only description applicable at this stage is the third. Thus by knowing more and more elements of an extensional description, we can decide between competing intentional descriptions. If the extensional description is the same, there is no means of doing so.

The paths of motion are an intentional description of the successive positions of a body. These positions themselves are given by an extensional description. The paths of motion form a geometrical picture. Many geometrical pictures — intentional descriptions — tally with a given extensional description of the successive positions of a body. Sometimes we can choose between the many geometrical pictures by knowing more positions of the body. This is possible in the case of the train and the station as explained before, but it is not always possible. Where it is not possible our choice of a particular geometrical picture will depend on the grounds of simplicity in the many senses explained before.

Upshot :

The many senses of simple have been discussed and it has been shown that a simpler description may not be a truer description. In such cases, simplicity has more of an aesthetic than a scientific value. But there are senses in which simplicity is not irrelevant to truth.

VIII

The Reality of Space

We have defined the motion of a body as its occupation of different places at different times. This definition assumes that (1) there is such a thing as Space, in some sense, to which the place belongs and (2) there is such a thing as Time in some sense, to which the different times belong.

But Philosophers have very frequently denied the reality of both Space and Time. We cannot therefore maintain our definition of Motion, unless we save the reality of Space and Time by meeting the arguments of these Philosophers.

Zeno's Argument against Space :

Let us begin with Zeno's famous argument against space. It runs as follows :—

"If all being exists in space, space as being must exist in a second space, and this in another, and so on ad infinitum"[47] (p. 53).

Aristotle replies to this by maintaining that "there is no reason why the first place should not be in something else.....just as health exists in warm beings as a state, while warmth exists in a material body as an affection, and so on indefinitely"[47] (p. 54).

Ranade has discussed Zeno's argument and Aristotle's answer to it. This discussion is rather puzzling as will be illustrated by the following remarks in it.

(1) " According to Aristotle, the regress involved in the argument is not of an objectionable kind"[47] (p. 53).

(2) " In the regress suggested by Zeno, the very meaning of the successive propositions is in question; on the other hand, in the regress suggested by Aristotle, the meaning of the propositions is quite definite"[47] (p. 54).

(3) " Hence.....by recognising that not all infinite regresses are objectionable, Aristotle gives a very clever answer to the sophism of Zeno"[47] (p. 54).

Now if it is to be maintained that Aristotle has successfully and not merely 'cleverly' answered Zeno, it will have to be supposed that Zeno's regress is as unobjectionable as Aristotle's. But Ranade maintains in the second statement, that Zeno's regress is vicious while Aristotle's is permissible. One certainly cannot get out of a vicious regress by pointing out that there are other regresses which are not vicious. One cannot cure a sick man by telling him that there are healthy people in the world!

One may get over Zeno's regress by denying that "all being exists in space". The joy one gets in hearing a good news, cannot be said to have *spatial* length or breadth or depth. My joy exists in me and it may be said that to that extent, it has spatial location, but this assumes that there is only one kind of "inness", the spatial inness. Pleasure is a state and it exists "in" me in the sense in which heat exists in hot water. The sense in which heat is "in" the water is not the same as the sense in which, the water is "in" the kettle. The difference is obvious from the fact that when pleasure is in me, I can be

said to be pleased, and when the heat is in water, the water can be said to be hot. But when the water is "in" the kettle, it cannot be said to be watered. The water cannot be predicated of the kettle.

But even after denying that every being exists in space, an important element of truth remains in Zeno's argument *viz.*- that "every being that can be spatially inside another, exists in space". Now space surely is spatially inside a larger space and this involves an infinite regress. But this regress is not vicious, as it only asserts that space is infinite in extent.

There is a difference in the sense in which the water is "in" the kettle, and a smaller space is in the larger. The water is not a part of the kettle, in the sense in which the inner surface of the kettle is part of the kettle. But a smaller space is a part of a larger space in this latter sense.

The difference in the two senses lies in the fact that water and the kettle are different objects, capable of existing independently, but the inner surface of the kettle cannot exist independently of the kettle.

Now common sense supposes that objects exist in space, just as the water exists in the kettle. In other words, there can be space without objects and objects without a particular space. The kettle need not contain any water, and water need not be in a particular kettle, though it must be in some container.

Spatial inness conceived in this sense, does involve a vicious infinite regress of the kind that is involved in postulating a serpent for supporting the earth. That every object having spatial properties, is inside some other object — in the sense in which the water is in the kettle — is not a fact of observation. A commet travelling beyond the atmosphere is not "contained" in anything. That it must be regarded as "contained" *in* space is thus a mere postulate based on the preconceived necessity of the container — contained relationship. If the necessity is granted

space must be regarded as being contained in another space, and the infinite regress follows. Granting the necessity and denying the regress, is surely hunting with the hound and running with the hare.

Bradley's Arguments against Space :

An argument similar to Zeno's had been advanced against space, by Bradley. It runs—

"Space to be space must have space outside itself. It fore ver disappears into a whole, which proves never to be more than one side of a relation to something beyond"[6] (p. 32).

"Space is a relation between terms, which can never be found"[6] (p. 32).

The dictum that there must be space outside space arises, I think, from mixing up two views of space, *viz.*- (1) Space as a set of properties possessed by objects and (2) Space as emptyness or nothing. Space includes all objects. The phrase, the "limit of space" therefore suggests that there is nothing beyond this limit. Since nothing in its turn is taken to mean empty space, it follows that there is space beyond the limits of space.

If we keep the two meanings distinct, the dictum does not follow. If there is no space apart from the spatial properties of objects, the phrase "limits of space" would mean, the boundary of the last object. Since there are no objects beyond this boundary, there could be no space, *i. e.* spatial relations of objects beyond this boundary.

If space means emptyness or nothing, the phrase "limits of space" would mean limits of nothing and it will follow that there is something and not space equated with nothing, beyond this limit.

Bradley is further intrigued by the fact that...."space must consist of extended parts, and these parts.....are spaces.....space is nothing but a relation of spaces", but

"space.....is evidently more than a relation..... The mere fact that we are driven always to speak of its parts should be evidence enough. What could be the parts of a relation?"[6] (p.31).

Space is certainly not a mere relation. It is points plus the relations between them. A line does not consist of points alone. A point bears a relation to another point, it is to be the right of or to the left of another point. It is not proper to speak of either points or their relations as parts of the line, in the same sense in which half the line is part of the whole line. A line is like a queue. Half of the queue is a queue. But the persons standing in the queue are not queues. The queue is neither a collection of persons, nor merely a relation between persons, it is persons standing in a definite relation to one another. To describe space as a relation of spaces is as wrong as to describe a queue as a relation of queues.

Bradley further argues that....." any space must consist of parts; and if the parts are not spaces, the whole is not space"[6] (p. 31). This is not quite correct. Let us again take the example of the queue. A queue may consist of two persons only, and in that case the queue has no part which is a queue. The persons are not queues but the whole formed by them is a queue. The principle that any queue must have a part which is also a queue will require an assumption that the queue, like a geometrical line is infinitely divisible.

On the assumption that space is infinitely divisible, Bradley argues that we are not able to find the terms, *i. e.* the points between which the spatial relations hold, since the points can be obtained only after the infinite division is completed. "Space is essentially a relation of what vanishes into relations, which seek in vain for their terms. It is lengths of lengths of nothing that we can find"[6] (p. 32).

Again the example of the queue helps us here. We do not have to single out persons in the queue in order to find out the relation according to which the queue is arranged. A

back portion of the queue has the same relation to a front portion, as a person in the back of the queue has to a person in front. Similarly it is not necessary to single out the points of a line to find out what relations they bear to one another. Parts of the line can serve the purpose. Just as a point can be to the right of another point, a part of the line can be to the right of another part.

Bradley's objection to the identification of space with the spatial properties of objects is that, on this view the properties such as extension will have to "in-here" in the objects and "inherence" is "in principle unintelligible"[6] (p. 33).

The unintelligibility is illustrated in the following way. " A thing is not any one of its qualities, if you take that quality by itself; if sweet were the same as "simply sweet" the thing would clearly be not sweet"[6] (p. 16). What seems to be meant is that if sweetness is the only quality which sugar has, and sugar is identical with that quality, the statement sugar is sweet would mean that sweetness is sweet. But sweetness can no more be sweet than bravery can be brave.

The argument ignores that "sugar is not sweetness" but "sugar is sweet". Sugar is identical with an instance of sweetness, not with sweetness, and an instance of sweetness is certainly sweet. An act of bravery is an instance of bravery, and the act can certainly be described as brave. If sugar has no quality besides sweetness, sugar is nothing but an instance of sweetness, and the proposition "sugar is sweet" amounts to saying that "an instance of sweetness is an instance of sweetness". This is a tautology but not a contradiction.

Bradley complains : "If you predicate what is different, you ascribe to the subject what it is not; and if you predicate what is not different, you say nothing at all"[6] (p. 17).

This complaint is unfounded. If sugar is nothing but an instance of sweetness, the ascription of the quality to it amounts to a tautology, but it is an instance of sweetness

as well as whiteness and such other qualities. Saying that sugar is sweet can therefore be interpreted to mean than an instance of whiteness is an instance of sweetness. Though whiteness is not sweet, what is white can be sweet. Bradley's objection to this is that....."in so far as sugar is sweet it is not white or hard, for these properties are all distinct"[6] (p. 16). But "distinct" does not mean "incompatible". The class of philosophers is distinct from the class of women, but the same person can be a member of both these classes, and be a woman philosopher. Distinct classes can have identical members.

Upshot :

The reality of Space has been defended against arguments such as those of Zeno and Bradley.

IX

The Reality of Time

Bradley's Attack on Time :

It is now time to consider the arguments against the reality of Time. We may conveneintly begin this task with Bradley whom we left at the end of the last Chapter. His argument runs as follows :—

"If you take time as a relation between units without duration, then the whole time has no duration, and is not time at all"[6] (p. 33). The hollowness of this contention can be exposed by a simple example. There is a hierarchy of posts in different departments. These hierarchies differ in heights, in the sense that there are more posts from the lowest to the highest in one department than in the other. But though the hierarchy has a height, the posts themselves have no height. (They are *at* a height but they themselves are not tall or short. A man flying at a height of 500 feet is not 500 feet in height). The posts may be regarded as units of the hierarchy, in the sense in which Bradley talks of units of time. The hierarchy thus possesses a property which the units do not possess. Similarly time can have a property like duration, even if its units do not have it.

In fact it is not the units of time but instants that are

durationless. Instants are the ultimate terms between which the temporal relations hold.

Many of Bradley's arguments against space and time are based on the time-honoured difficulties of infinity which can now be regarded as solved. The argument about the discontinuity of time[6] (p. 36) is essentially similar to Zeno's argument of the stadium and the solutions of the latter are applicable to it.

In recent times McTaggart has advanced some arguments against time which are independent of the difficulties of infinity. I will now proceed to consider them.

McTaggart's Attack on Time :

McTaggart finds two types of relations in time (1) earlier than, later than and (2) past, present and future. He regards the former as permanent, but not the latter. "If *M* is earlier than *N*, it is always earlier. But an event, which is now present, was future, and will be past"[35] (Vol. II, p. 10).

This version is not acceptable. There is a third relation in addition to earlier than, and later than, *viz.*- simultaneous with. An event *e*, simultaneous with an event *c* is present in relation to *c*. Events earlier are past and later are future. Thus past, present and future are as permanent, if at all, as earlier, simultaneous and later from which they are derived. To argue on the lines of McTaggart, if *M* is simultaneous with *N* it is always simultaneous.

When I say that a particular event is no longer present, what I mean is that it is no longer, simultaneous with the knowledge of the event expressed in the proposition. But it never ceases to be present to the earlier knowledge of that event with which it is simultaneous.

The relation of simultaneity is in one way different from the relations earlier than and later than. It is not asymmetrical. Thus unless we regard an event as simultaneous

with itself, we cannot find the relation of simultaneity in one series of events. We require an event outside the series. This outside event is cognition. An event is simultaneous with its cognition, and is said to be present to it. It is this usage of the term present, as essentially referring to simultaneity with cognition, that creates the impression, that past, present and future are essentially different from earlier, simultaneous, and later.

From the fact that the relation of earlier and later is permanent McTaggart argues that "*N* will always have a position in the time series, and always had one. That is, it always has been an event, and always will be one, and cannot begin or cease to be an event"[35] (p. 12). While drawing this inference, McTaggart forgets his earlier statement "*If M is ever earlier than N*, it is always earlier". Once *N* succeeds *M*, the fact *M* is earlier than *N*, cannot be altered, but *N* was certainly not earlier than *N* at the time *N* had not emerged.

If we wish to define time in terms of the relations earlier than etc., we must describe the state of affairs without using the word time. This can be done as follows :—

(1) "*M* is present to me" means that *M* is simultaneous with my cognition of *M*.

(2) If there is no event later than *M*, the future of *M* in non-existent.

(3) The proposition that there is no event later than *M* is true if it is simultaneous with *M*. (Assuming that the future is non-existent).

The fact that a proposition can be true only if it is simultaneous with something else and false otherwise, is no more strange than the fact that the proposition "My name is John" is true, if uttered by John but not otherwise. The meaning of a proposition depends not merely on words and syntax, but also on its context.

McTaggart points out that the characteristics of an event cannot change and infers that there cannot be any change or Time. The death of Queen Ann, is the death of Queen Ann. When it was in the future, it was not the death of some one else, and when it is past it is still described as the death of Queen Ann. No part of its description changes except for the words is, was and will be[35] (p. 13).

This argument assumes that the existent and the non-existent must have different descriptions. This is by no means the case. Merely from the description of a person, nobody can find out whether he is existent or non-existent. If it were so, controversies about the historicity of a person, would be impossible. His accepted description, by itself would be sufficient to decide the issue. An imaginary rupee has no quality more or less than what a real rupee has. The only difference is that an assertion of existence is false in one case and true in the other.

From the fact that an event simultaneous with its cognition is called present, and there could be no past and future without the present, it is argued that, the past, present and the future depend on 'consciousness'. "If there were no consciousness, there would be events earlier and later than others, but nothing would be in any sense past, present or future. And if there were events earlier than any consciousness, those events would never be future or present, though they could be past"[35] (p. 14).

This conclusion can be avoided by extending the use of the word present to all cases of simultaneity. Two simultaneous events may be regarded as present in relation to each other, even if none of the events is a cognition, or consciousness.

According to McTaggart a mere series of events, from earlier to later, does not amount to change. The fact that a poker "is hot at one point in a series and cold at other points cannot give change, if neither of these facts change and neither of them does"[35] (p. 15).

Viewed in one way, the argument is similar to Zeno's arrow. 'The fact that the arrow is here at one point in a series and there at other points, cannot give motion, if at neither point it moves. The answer is also similar. Change is nothing but a series of events from earlier to later.

Viewed in another way, the argument seeks to show that the events in the series have to be regarded as permanent, since what is earlier is always earlier. Permanent events bound by permanent relations can hardly be described as time.

But what is meant by permanent, and what is meant by time? If time is nothing but a series of events described by the three relations earlier, later and simultaneous, a permanent event would be an event which is simultaneous with all events. Since the heating of the poker is not simultaneous with all events, clearly it cannot be described as a permanent event.

This at once disposes off the example given by McTaggart. The proposition that "at S" (degrees of latitude) "the meridian of Greenwich is within the United Kingdom" is true, while the proposition 'at S' the meridian of Greenwich is within the United Kingdom is false exactly as the proposition "the poker is hot is true at one point of time and false at another"[35] (p. 15). Still the propositions about the meridian do not give us change, while the propositions about the poker do, bacause the meridian's being, and not being within the United Kingdom are simultaneous, whereas the poker's being and not being hot are not.

The judgements like "the battle of Waterloo is in the past" can be sometimes true and sometimes false. This judgement for example is true after the battle of Waterloo but was false before. Such judgements can be made precise by saying that "the battle of Waterloo is earlier than this judgement" etc. Thus made precise McTaggart argues "....all judgements are either always true or always false. Then I maintain no facts change"[35] (p. 16).

The principle that all judgements are either always true or always false cannot be accepted. There are judgements which are neither true nor false. Some judgements in a sense are neither true nor false. "Hamlet loved Ophilia" is neither true nor false. We cannot argue that since Hamlet did not exist, the judgement is false, because the falsity of the judgement implies that the judgement "Hamlet did not love Ophilia" is true. Judgements about the future may be said to belong to this category. On this point I agree with Broad.[7·1]

When literary critics discuss the question whether Hamlet loved Ophilia, what they are really discussing is whether the behaviour of Hamlet towards Ophilia in the play is (1) more similar to that of a lover or (2) to that of a non-lover. These two judgements can be true or false.

The notion that what we can talk about must exist or be real in some sense has created a good deal of confusion in philosophy. In order to get clear of this confusion, we must distinguish between (1) logical validity and (2) the truth of a proposition or a discourse. Logical validity can be ascertained by inspecting a piece of reasoning or a proposition. The validity of the proposition, "The square circle must be square" can be established by examining the proposition itself. But no amount of inspection of a proposition or a discourse can tell us whether it is true. The structure of true and false propositions is completely identical.

Another important point is that a proposition does not assert its own truth. In order that this be so, a proposition *P* and a proposition about the proposition *P* will have to be identical. A proposition may assert that John had two sons. But it cannot also be held to assert that the assertion is true. If we hold this, we make it impossible for the proposition to appear in a work of fiction.

The fallacy in the ontological argument is partly due to a confusion between logical validity and truth, and partly to a confusion between a proposition and a proposition about

a proposition. The statement that (1) the notion of perfection includes the notion of existence may imply the statement that (2) the perfect being exists. But implication only establishes validity and not truth. From the fact that the proposition 2 is implied, it cannot be inferred that the proposition 2 is true.

The reader may think that distinguishing between an assertion and its truth lands us in a vicious infinite regress, since the assertion of the truth will again have to be asserted to be true.

This fear is unfounded. From the fact that assertion of *P* and the assertion of the truth of *P* are different it, does not follow that the assertion of the truth of *P* and the assertion of the truth of the truth of *P* are not identical.

Two series?

McTaggart talks of two series, the *A* series and the *B* series. The *A* series is the series of positions which runs from the far past through the near past to the present and then from the present through the near future to the far future or conversely. The series of positions which runs from earlier to later, or conversely, is according to him the *B* series. I think this distinction is quite unwarranted and is responsible for most of the difficulties McTaggart projects on Time. Past, present and future are but other names for earlier, simultaneous and later, excepting for the fact that we use the terms past, present and future with reference to events in our own life.

It is obvious that a time-series need not have any relation to events in the life of a person. Even if there is no consciousness to know events in the world, they will still have a time-order. This is clearly exemplified by fictitious events which have a time order, though, since they are fictitious, have no time relations to events in our life. The adventures of Don Quixote form a definite time series. "The adventure of the galley slaves, for example, is later than the adventure of the Windmills"[35] (p. 16).

McTaggart explains this away by saying "Time only belongs to the existent. If any reality is in time, that involves that the reality in question exists"[35] (p. 16). This is certainly not the case. Time is like any other property, say, pink. It does not necessarily belong to what is existent. "Pink rats" are a stock example. The fact that the adventures of Don Quixote have a time-order is completely independent of the question whether they really took place.

McTaggart's difficulties about time arise from the curious supposition of his that "……no relations which are exclusively between members of the time-series can ever change"[35] (p.19). One would have supposed that change is the grandest title for anything to be called temporal. It is wrong to suppose that members of a time-series do not acquire new relations. To suppose that an event always has all its time-relations is to suppose that all temporal relations are wholly simultaneous. But this is by no means the case. In order to get this point clear, we need the following definitions :—

(1) *A* is wholly simultaneous with *B*, if it is simultaneous with *B*, and no part of it is earlier or later than *B*.

(2) *A* is partly simultaneous with *B*, if only a part of it is wholly simultaneous with *B*.

Let us take three events *A*, *B* and *C*, such that *A* is earlier than *B* and *B* is earlier than *C*. Now just as the events can be earlier, later or simultaneous, the relations earlier, later and simultaneous can themselves be earlier later or simultaneous. In the case of our three events *A*, *B* and *C*, the relation — say $R_2 - C$ is later than *B*, is wholly simultaneous with the relation — say $R_1 - B$ is later than *A*, because no part of it is earlier or later than R_1, and it is simultaneous with R_1. But R_1 is not wholly simultaneous with R_2, because a part of it is earlier than R_2, that part which is not simultaneous with the emergence of *C*. Generally later relations are wholly simultaneous with the earlier ones, but the earlier ones are only partly

simultaneous with the later ones, since only their later parts are wholly simultaneous with the later relations. This is the sense in which time relations are not permanent.

Many unnecessary difficulties arise as a result of ignoring this principle. For example, Hoskyn says that if nature is regarded as a geometrical system, it will have to be regarded as static[22] (p. 342). This statement is based on the view that the relations studied by geometry are applicable only to space. It has been shown that this is by no means the case (Chapter IV). The difficulty in conceiving time in terms of geometrical relations lies in the inveterate habit of supposing that the terms and relations about which we speak must all coexist and exist at the time we speak about them. Some of the relations we speak about may not yet have come into being and some of the terms we mention may have ceased to exist. But as long as we can speak about them, they can form part of a description which though geometrical, is not static or spatial.

Deshpande's Defence :

Even those who uphold the reality of Time against McTaggart's arguments are prone to regard time-relations as permanent or timeless, and are afraid that representing Time as a series of events robs it of its mobility. Their argument is the following :—

……"The proposition '*M* is earlier than *N*' states a timeless fact……the copula 'is' cannot be replaced by the past and future terms of the verb to be *e. g.* we cannot say '*M* was earlier than *N*' for this would imply that it is not now earlier, and this is impossible. Similarly we cannot say '*M* will be earlier than *N*' for this would imply that it is not now earlier, and this too is impossible. It seems obvious therefore that the copula 'is' in the proposition '*M* is earlier than *N*' is not a temporal 'is' copula as in 'it is raining', but a non-temporal one as in 'Twice two is four' "[15.1] (p. 84).

These examples are not decisive. We certainly say "the Vedic Sanskrit *was* the earlier form of Sankrit". If both the Vedic and classical Sanskrits were spoken to-day, we would perhaps have said, "the Vedic Sankrit *is* the earlier form of Sanskrit". The expression "the Vedic Sanskrit *was* the earlier form of Sanskrit", in case both the Sanskrits were being spoken, would be justified on the ground that, the earlier prevalence of the Vedic Sanskrit and the later emergence of the classical Sanskrit are events belonging to the past. Even if two brothers are living, we may say, *A was* born earlier than *B*, instead of saying *A is* older than *B*. Being older than is the same as being born earlier, but it directly refers to the person who is alive, and to his birth only by implication. The births of the persons and not the persons are events in the past, and the use of was is justified in their case. In the case of living brothers, no one would say "*A* was the elder brother of *B*". But in the case of the Pandavas, we do say "Dharama *was* the elder brother of Arjuna".

"*M* will be earlier than *N*" is also a permissible statement. For example "Tea will be earlier than breakfast" means nothing but that ".....the serving of the tea will be earlier than the serving of the breakfast". If tea has already been served we may not say "tea will be earlier" but we can still say "breakfast will be later". This amounts to saying "*N* will be later than *M*", which cannot be unless "*M* will be earlier than *N*" is true. The futurity which belongs to the former relation, therefore, automatically belongs to the latter.

Even if tea has been served, we can say "tea will be followed by breakfast". "Being followed by" and "being earlier than" are the same relations, but the absence of passive voice in 'Tea will be earlier than breakfast' suggests that the earlier serving of the tea, will come later than the time at which the proposition is made, whereas the passive voice in "followed by" leaves the tea inactive and ascribes the act of coming later than the time at which the proposition is made, to the breakfast.

These examples show that we do tend to regard the relations of the objects past as past, of the present as present and of the future as future. Linguistic usage provides no evidence for the view that time-relations are timeless.

It is often said that propositions ascribing a date to an event, for example the statement that India became independent in 1947, are true at all times. But there is no reason to suppose that we must regard the proposition as true in 1946, even if we change the phrase "became independent" to "will become independent". Even if we suppose that some temporal characteristics are timeless, there is no contradiction involved. A temporal characteristic can be timeless just as a numerical characteristic can be numberless. The proposition "The Apostles are twelve" ascribes a numerical property to the class of Apostles. But this numerical property itself *viz.*- twelve, is numberless. Twelve is a number but it *has* no number.

That time relations are not "eternal" as is alleged so often is enough to establish that past, present and future are not merely subjective. But the fact remains that the past, present and future can be completely analysed in terms of earlier, simultaneous and later.

Deshpande holds quite the opposite view. According to him....."far from pastness, presentness and futurity being analysable in terms of the relations of earlier and later, these relations themselves imply pastness, presentness and futurity. To be convinced of this, it is only necessary to ask what is meant by saying that *A* is earlier than *B*.....it means simply that A *happened* earlier than *B*, *i. e.* the concept of being earlier implies the concept of happening. To say that *A* happened earlier than *B* means nothing else than that when *A* happened *B* had not happened, and when *B* did happen *A* was no longer happening"[15.2] (p. 86).

One may ask here "what is meant by the phrase '*When A* happened'?" We can only define this 'when' as the class of events simultaneous with *A's* happening. Thus Deshpande

has not succeeded in analysing simultaneity in terms of happening. This is further clear from his definition "To happen is to be present". If this is the meaning of "to be present", it should be meaningless to ask whether a happening is present or past. But this is by no means the case. Further we should be able to substitute the derivatives of the word happen whenever we use the word present. But this is not possible. "The present President of India", as far as I see, can be translated in terms of "happening" as "the President of India whose presidentship is happening or occurring". Now "is happening" is present tense and to say that "*A* is happening" is using a synonymous expression for "The happening of *A* is present". Giving synonyms is not analysis. "Bigger means greater" is not an analysis of the concept bigger.

The phrase can, on the other hand, be easily analysed in terms of simultaneity. "The present President of India" is the president whose presidentship is simultaneous with the occasion at which the phrase is uttered.

There is a sense in which the present cannot be translated by "simultaneous", the sense in which the present is the opposite of absent. In this sense what is present is not necessarily present to some one, not even to itself. Presence is not a relation, not even a reflexive relation. What is the connection of this presence with the presence as a time-relation?

I think the word present used in the above non-relational sense is simply a synonym for existent or real, and has no temporal connotation. *A* may be present at a particular time, just as *A* may be hot at a particular time, but this does not imply that existence is a temporal property, any more than hotness is. It is felt that existence has an intimate and inseparable connection with the temporal present, because in the English language the word "is" indicates both, existence and temporal presence.

McTaggart persistently ignores that an existent event and a non-existent event, can be completely identical excepting

with regard to existence. This is clear from the following attempt of his to deduce the reality of the future from determinism. "If X intrinsically determines a subsequent Y,.....it will be true that, since there is an X, there must be a subsequent Y"[35] (p. 24).

This is best discussed in connection with astronomical predictions. "A total eclipse of the sun visible in Cornwal is prophesied for the 11th August 1999[16]" (p. 297). The prophecy is a statement about the future. In what sense is it true?

It is certainly not true in the sense that the existence of the eclipse is simultaneous with my writing this statement. We regard to astronomical predictions as true because they state a logical implication between the present and future positions of the heavenly bodies. One term of the implication *viz.*- the present positions, is real, while the other *viz.*- the future positions, is not.

Logical implication does not depend on the reality of either or both of its terms. The implication stated by the statement "If it rains on the Sahara, the Sahara will be fertile", can be real even if "the rain on Sahara" and "the fertility of Sahara" are as fictitious as Don Quixote's adventures.

But it is not so with some other relations. The imaginary events in a historical novel or the nineteenth century are earlier than the second world war. But this relation of earlier than should not be regarded as real, since it is a relation of unreal events.

McTaggart after raising a problem as to how *M* can be past and present as well, solves it himself by pointing out that ".....*M* was past for *X* and present for *Y*....."[35] (p. 28). But he thinks this makes the past and the present subjective. One would say this makes it relative and not subjective. *E* may be a daughter for *X* and wife for *Y*. But this does not make daughters and wives subjective! Just as the

same person can have different family relations to different persons, the same event can have different time relations to different events.

Shriharsha's "Refutation" of Time :

Shriharsha in his *Khandana* has advanced some arguments against Time. He begins by asking what is meant by "the present time". The present time is sometimes defined as the property of its own cognition. When a time is cognised as past, the cognition does not have the property of pastness, similarly the cognition of the future is not itself future. Thus presentness is that property of time which is common to the cognition and its object*.

Shriharsha criticises this definition as follows. Cognition can be regarded as self-luminous or non-self-luminous. Self-luminous cognition can cognize itself, while non-self-luminous cognition cannot.

Now if cognition is non-self-luminous, presentness cannot be known, for the cognition whose property it cannot know itself. If another cognition is said to know it, this other cognition can only be before or after the first one. In any case, it can know the first cognition when it no longer possesses the property of presentness.

If cognition is self-luminous, it has the power of knowing itself, apart from this power, its other propertes are just the same as those of the non-self-luminous cognition. Even non-self-luminous cognitions can know the properties of other cognitions. So the problem remains. If cognition C_2 cannot see the property of presentness in cognition C_1, how can C_1 see it in itself? The mirror gives me the power to look at my own face, but it does not give me the power of seeing what others cannot see there.

---

*ग्राहकविज्ञानविषयो ग्राहकविज्ञानाश्रयश्च कालो वर्तमानः ।[50] (p. 675)

An Answer to Shriharsha :

The whole confusion arises from supposing that presentness is a property, which belongs to an individual cognition. In fact it is a relation, the relation of simultaneity between the cognition and its object. Since simultaneity is symmetrical, the cognition and its object are simultaneous with each other and the relation is identical for both. This is the truth in the assertion that presentness is a temporal property, common to the cognition and its object. When the cognition (or rather recollection) is of the past, it is later than its object and the object is earlier than its cognition. Thus the relation is not identical for both.

Self-luminous cognition can cognise itself as present because, simultaneity like self-cognition is reflexive, and the cognition is simultaneous with itself. Cognition C_2 cannot cognize the cognition C_1 as present, because C_2 is not simultaneous with C_1.

Many arguments against Space and Time have been surveyed and answered so far. It is neither possible nor necessary to consider all such arguments. In essence, they challenge us to state our experience of Space and Time precisely and consistently and enough has been said here to indicate that this is possible.

The relevance of this discussion to the problem of motion needs no justification, because motion is both spatial and temporal. The arguments of McTaggart are even more directly connected with motion. The different positions of a moving body are a series constituted by time-relations such as earlier and later, past and present, etc. and McTaggart's whole polemic is against such a series. By answering this polemic, we have cleared the way for the possibility of motion.

Upshot :

Bradley's objections against Time are mainly concerned with divisibility and have been answered.

McTaggart's contention that time-relations are permanent cannot be accepted. His contention is based on the assumption that what we can speak about must be present.

Shriharsha's argument against Time is based on a confusion between a property and a relation.

X

Points and Instants

We have seen that Space and Time cannot be argued away and their reality has got to be accepted in some sense. But in what sense exactly must we admit Space and Time? Is there space without spatial objects and Time without change? Russell, in his Principles of Mathematics argues that there could be a pure space consisting of points. This view can be very easily made applicable to Time also. I will now proceed to examine it.

The main difficulty in this view is that, we cannot say how a point differs from other points. The only difference is that of position. But the position of one point is defined by means of other points. One point is so many inches to the right of another. This is a relation between two points. Apart from such relations, do the points have any properties?

According to Russell two things may be simply different without having different properties, for the very statement *A* and *B* have different properties is not possible unless *A* and *B* are already different. "Before two subjects can differ as to predicates they must already be two"[48·4] (p. 452).

But one may similarly argue that the proposition *A* and *B* are identical is not possible unless *A* and *B* are different. "Before

two subjects are" identical "as to predicates they must already be two".

This second argument throws light on the weakness of the first. The assertion *A* and *B* are identical is certainly legitimate and is frequently made. In saying "My wife is the sister of Mr. *X*", I am identifying the two, *viz*.- my wife and the sister of Mr. *X*. I am not saying my wife is my wife.

In order to affirm identity or difference, we need two names or two descriptions, but not necessarily two things that are described or named. Thus from the mere fact that two terms are needed to assert any difference, it cannot be concluded that the things named by the two terms would be two even if all the differences between them that can be asserted were not there.

Thus we cannot prove that there can be differences between points of such a nature that we cannot describe what those differences are. It may not be a neat expression to say that two points must have different properties or "predicates". But it seems a legitimate demand that if there are points, apart from objects, they must differ not merely in position, but in some other respect also.

And not merely should they differ, but we should be able either to perceive or at least be able to describe that difference. The examples cited by Russell fulfil both these conditions and are not therefore analogous to points. We can directly perceive that red and blue are different, but we cannot perceive any difference in two points apart from their position. Russell's example of the lecturer with a bad memory for faces also does not help us here. Suppose in the first row in a lecture hall there are only two seats. The lecturer has such a bad memory for faces that he can never say whether the two gentlemen that are occupying these two seats to-day are the same as those that were sitting there yesterday. Even such a lecturer will be able to know that there are two faces in the front row, and not one.

Similarly, "among simultaneously presented points, it is easy to distinguish...." though by taking the points separately we may not be able to say in what way one differs from the other. So runs the argument[48·4] (p. 452).

But the analogy is not quite convincing. Even the lecturer with a bad memory for faces can see the different characteristics of the faces when they are before him. He can even describe those characteristics to others. He will not say that he sees no difference between the faces excepting that they are in different positions.

But it is quite otherwise with points. Is it "easy to distinguish" even among simultaneously presented points? In fact points as such are never presented, only objects consisting of points are presented. In the case of the faces, it cannot be said that they are not presented even when they are before the lecturer. Owing to his bad memory the lecturer may not remember them, when they are out of sight and may not recognize them when they are presented again. But he will never say that the faces are not presented at all, only the persons to whom they belong are presented.

Russell further observes that the differentia of points lie also in the different relations they have "to the objects in them". "Thus they seem to be in the same position as colours, sounds or smells. Two colours or two simple smells.....have, like points, different relations to other terms"[48·4] (p. 452).

The statement that points differ "in their relations to the objects in them" is capable of two interpretations : (1) If an object O_1 occupies a point P_1 and an object O_2 occupies a point P_2, the relations of O_1 to P_1, and of O_2 to P_2 are different and (2) the relation of P_2 to O_1, is not the same as the relation of P_1 to O_1 and vice versa.

In the first case, it will be more correct to say that the points P_1 and P_2 have the same relation to different objects, rather than that they have different relations. We say that the

number of eyes a man has is the same as the number of legs he has. Eyes are different from legs, but their number is not. Similarly two pairs of terms, may be different but the relation which constitutes the pairs may be identical. Orange is red and indigo is blue. Orange is not indigo, nor is red blue. But the relation of red to orange is the same as the relation of blue to indigo. In the case of the points, P_1 and P_2, O_1 occupies P_1 and O_2 occupies P_2. All the terms O_1, P_1, O_2, P_2 are different but the relation of occupation is identical. Thus it cannot be said that points have different relations to the objects in them.

On the second interpretation, the two points P_1 and P_2 may be said to be different because P_2 is not related to O_1 in the same way as P_1. This is the case when O_1 occupies P_1 which is earlier than P_2. But this is purely an external relation. Though P_2 does not now have the same relation to O_1 as P_1 has, it may have it later on without undergoing any change in itself. This will happen when O_1 moves to P_2. When a dead runner's speed record is broken, he acquires a relation he did not have before, *viz.*- that of being the second fastest runner, without undergoing any change. If the relations with regard to which P_1 and P_2 are different are of this purely external nature, P_1 and P_2 cannot be said to be intrinsically different simply because they have such relations.

It is obvious that if there are points apart from and independent of objects, they must have other properties besides having relations to other points, or in other words it must be possible to mention the points without mentioning their relations. Unless it is possible to name A and B, it is not possible to say that they are related. If the only possible way of naming A and B is by saying that they are terms of the relation R, then the relation will obtain only between these two terms, and there will be no other instances of the relations. If the points P_1 and P_2, have no other characteristics but that they are related by the relation to the right of, then of no other points

should it be possible to say that one is to the right of the other. This is clearly absurd.

"But you are ignoring", it may be pointed out "that a point is nothing over and above all the relations it has, though it is certainly more than only one of its relations. A point has other relations besides being to the right of another point and therefore being to the right of does not give its complete description. No two points have completely identical relations. In fact two points are only two sets of relations. It does not follow that the relation " being to the right of " is possessed by only one point, because this relation may be common to many sets of relations. Since sets of relations are points, it follows that there can be many instances of the relation to the right of.

"The case of points being nothing but a set of relations is analogous to the case of substance being nothing but a set of attributes. If a substance has only one attribute, it does not follow that no other substance can have that attribute, even if we suppose that a substance is identical with its attributes. Even if a red substance is identical with the quality red and it has no other quality there can still be other instances of the quality red. A jar is identical with itself. A jar is nothing over and above being a jar. But it does not follow from this that there is only one jar."

To this we reply as follows :—

The analogy with substance and attributes is false. From the principle that substance is indentical with its attributes it does not follow that one substance is identical with another. But from the principle that a point is identical with its relations, it does follow that one point is identical with another point. Excepting reflexive relations like identity, a relation will require more than one point. Now if a point is identical with the relation, it will be identical with all the points that have the said relation. The affirmation of a quality of a substance, involves no reference to other substances, but the affirmation of a non-reflexive relation of a term, does involve reference to other terms. Relations are therefore not analogous to qualities.

By now it is fairly clear that if there are points apart from objects, they must have characteristics other than relations with each other, or relations to the objects. It is not possible to imagine what these charateristics could be. We must then do away with the necessity of postulating such featureless entities.

This brings us to the theory of space as a set of relations between perceptible objects, and not something independent of objects, *in* which the objects reside. That some such theory of space is necessary can be easily realised by sailors, sailing on a boat without the mariner's compass in a cloudy night. Under these conditions, the relations, east, west, etc., which are assumed to be inherent in space, at once appear to become meaningless. We cannot possibly think of spatial relations in pure space.

The main difficulty in accepting the relational theory of space, is that, it seems easily possible to imagine empty space, where there are no objects, and the relational theory seems to have the burden of proving that there is no such space. But in fact what is imagined to be empty space, is only distance between objects. Now distance is a relation between objects which cannot be defined without having recourse to entities other than pure space. A distance between two objects is either defined by a third object, whose two ends are spatially identical with *A* and *B* respectively, or by a chain of events starting from the one and ending in the other such as emission of light by *A*, and receiving of it by *B*. This chain may consist of two events only, one on *A* and the other on *B*, with no events in between. What is necessary is that the two events should not be independent, but causally connected. When we talk of distance between two objects, we are necessarily talking of something connecting the two objects, not of the "amount of nothing" between them.

In fact, no precise meaning can be attached to distance in empty space. A line in empty space consists of featureless

points. Since the points are featureless, all points are indistinguishable and we cannot meaningfully talk even of the first and the last point on the line, not to speak of the distance between those points.

On the whole, pure space should be rejected on two grounds (1) there is nothing analogous to it in experience and (2) there is nothing in experience which compels us to postulate it.

The relational theory is as valid with regard to time as to space. Just as there is no need to postulate points apart from objects, there is no need to postulate instants apart from events.

Magnitudeless Points and Durationless Instants :

If we thus get rid of pure Space and pure Time, what status are we to accord to the points without magnitude and the instants without duration, of which the Mathematician speaks?

Whitehead[58.2, 58.3] has shown how points and instants can be difined in terms of finite volumes and durations. Points and instants on this view, do not belong to the minimum vocabulary required for describing Time and Space. They can be "constructed" from perceptible entities. Russell[48.3, 48.4] and Broad[7.1] have explained this procedure with admirable clarity. I do not have anything to add on this score.

But I wish to draw the attention of the reader to some logical considerations which will serve to reduce the halo of mystery that has gathered round points and instants.

The conception of a length without breadth and derivativity of a point without magnitude, appears perfectly commonplace, when we consider non-spatial lines and non-spatial points.

A line of Kings obviously has no breadth. Only a very obtuse person could suggest that the fatness of the Kings

constitutes the breadth of this line. The Kings form a line in virtue of the relation of succession. Any property logically independent of this succession is irrelevant.

The Kings themselves are the "points" on this line. These points obviously have no length. The length of the line of Kings depends on the number of succession relations. But a King by himself cannot have succession. Succession is a relation between Kings. It is not the property of an individual King.

Now suppose there are two lines of Kings. Only one of the present descendants has a son. The two present descendants enter into an agreement that, the one without a son should adopt the son of the other and if this son has two sons, one of them will belong to the one line and the other to the other. It luckily comes about that the adopted one has two sons and the two lines continue again.

Here is a case of an intersection of two lines. The adopted issue is the common "point" in the two lines of Kings. He belongs to both the "lines". This point can obviously have no breadth or length. For if it has breadth or length it will be common to the two lines, since the point is common to the two lines. But the two lines contain nothing excepting the Kings and the relation of succession. Succession is a relation between two Kings and since no two Kings are common to the lines, no succession is common to them. The only thing common to them is a single King, and not a relation. Since it is some relation like succession which alone could constitute the length or breadth of the point, the point has no breadth.

This illustrates, how a point has neither breadth nor length.

The essential principle to grasp in this connection is that of logical independence, not of non-existence. The line of Kings has no breadth, because we are only considering a succession. The properties which are not logically implied by succession, may be existing in fact, but they are irrelevant

in a discussion of succession. A line has no breadth in the same sense in which a Mathematician has no sex. The person who is a Mathematician certainly has some sex. But the sex of the person is logically unrelated to the person's role as a Mathematician. Similarly any actual line drawn on paper must have some breadth. But the breadth is logically independent of its length. It is neither increased nor decreased by increasing or decreasing the length.

If someone insists that a line has breadth, though this breadth has no minimum, and must not be ignored, it will be impossible to talk of one line. Instead of talking of a line *AB*, we shall have to talk of a class of infinite number of parallel lines having identical properties. Apart from the substitution of this phrase our geometrical statements will be unchanged, and no logical purpose will be served by the introduction of this phrase.

If we talk of a square, we must talk of an infinite number of identical squares, formed by displacing the same square horizontally and vertically. If on a black-board we move a square block to the right, the different positions of the vertical sides can form the breadth of the vertical sides. Similarly, by moving the block up and down the different positions of the horizontal sides can form the breadth of the horizontal sides. Thus all the four sides have breadth. But the breadth of the sides only represents different positions of the same identical square, or in other words if lines without breadth form one square, lines with breadth will form an infinite number of identical squares. We have therefore gained nothing by insisting on not ignoring the breadth of the sides beyond using this cumbrous phrase "infinite number of squares identical with the square *ABCD*" in place of a "square *ABCD*". The use of this cumbrous phrase makes no difference to our geometrical statements.

Once it is realised what is meant by the statement "A line has no breadth", there is no difficulty in understanding how a

point cannot have either breadth or length. Two straight lines can intersect. Intersection is only a common part of the two lines. Now this common part cannot consist of breadth, since the lines themselves have no breadth. It cannot consist of length either, because both the lines are straight lines and if they are identical for a part of their length, they must be identical throughout. A straight line maintains the same direction from beginning to end and therefore the direction of any of its parts is identical with the direction of the whole line.

A magnitudeless point in this context stands for the common part of the two lines whose breadths are uncorrelated with their lengths, and are therefore logically irrelevant.

If we insist on not ignoring the lengths of actual points, we may treat the points *A* and *B* as equal circles. Now suppose the distance between *A* and *B* is to be ascertained. As Karl Pearson has pointed out, the distance betweent the circles *A* and *B* is an ambiguous concept, because distance between the different portions of the circles will be different[44] (p. 169). The right hand portion of the circle *A* will be nearer to the left hand portion of the circle *B*, than to its right hand portion (assuming that *B* is to the right of *A*). Now if we take a scale whose end portions just touch the right and left portions of *A* and *B* respectively, it will fail to reach *B*, when it is placed on the left portion of *A*. A scale which is long enough to touch the right portion of *B*, even when it is placed on the left portion of *A*, will extend beyond *B*, if its one end is placed on the right portion of *A*. But if we take a scale whose end portions just touch the central portions of *A* and *B*, we shall find that when placed on the left of *A* it touches the left of *B*, and when placed on the right of *A* it touches the right of *B*. In other words if it touches the one, it is capable of touching the other, and this touching occurs on corresponding portions of the circle. We may therefore regard this scale as the distance between the the circles. If we do not wish to consider the other distances, portions of the circles *AB*, which give rise to the other distances

can be regarded as non- existent. This amounts to treating the circle as a point.

Thus when we are speaking of a distance between two points, taking account of the magnitude of the points and regarding them as circles, only introduces innumerable other distances which are logically irrelevant.

Lengths without breadth and points without magnitude are essentially logical abstractions. A logical abstraction consists in "abstracting" a property about which we wish to speak, from all other properties which are logically independent and therefore irrelevant to it. This procedure is very commonplace and there is nothing mysterious about it.

Upshot :

It is both difficult and unnecessary to postulate a pure Space and Time. A relational view of both these can be satisfactorily worked out. The many difficulties about magnitudeless points and durationless instants are mainly due to a failure at logical abstraction.

XI

The Reality of Motion

(1) The Dichotomy.

In order to preserve the reality of motion, we resolved the arguments against the reality of Space and Time. We must now address ourselves to the task of resolving arguments which are directly levelled against the reality of motion.

The name of Zeno is most famous in this connection. He has advanced five arguments to prove the unreality of motion. Opinion perhaps was never so divided as about the value of these arguments. Most philosophers have described the arguments as "sophisms", Peirce even describes one of them as a 'rediculous little catch'[5] (p. 109). On the other hand hand, Hegel[21] Russell [48·3, 48·4], Ryle[49], etc., regard the arguments as immeasurably subtle. But sophistical or subtle the arguments have not failed to provoke "refutations", and even among those who describe the arguments as silly or sophistical there is no agreement about the exact nature of this sillyness or sophistry.

My interest was first aroused in these arguments by a line in Nagarjuna's[42] Madhyamik Karika which reads 'Movement cannot take place on the path which a body has traversed nor on the path which it has not traversed. Therefore a body cannot move.' After further reading philosophy, I

came across Zeno. The above is also one of his five arguments. Nagarjuna (A. D. 150)[41] (p. 205) is presumably later than Zeno, (5th Century B. C.[8]), (pp. 310-311), but this does not lead to the inference that he did not independently think out the argument himself. Nagarjuna has advanced other arguments against motion, besides the five arguments of Zeno, and these will also be considered in the sequel.

Whatever be the historical connection between Zeno and Nagarjuna, their arguments against motion made a profound impression on me, and it has lasted ever since I read them. I have always felt that dubbing the arguments as "sophisms" is an intellectual escapism, and a philosophy which treats them lightly is at least incomplete in a vital respect.

When I was in such a state of mind, it was a great joy to have been introduced to the works of Bertrand Russell, who not only attaches due importance to the arguments of Zeno, but has also satisfactorily solved many difficulties, which used to puzzle me. Russell claims that the problems raised by Zeno now belong to history and are no longer problems. He is also sure that a person adequately trained in Mathematics and Logic cannot fail to be convinced by his solutions, if he takes the trouble to study them[48·3] (p. 164).

This last claim does not seem to be supported by facts. Whitehead, whom nobody can charge of "inadequate mathematico-logical training", holds that even after making allowance for Zeno's mathematical errors a substantial amount of valid argument remains[58·1] (p. 106).

As for philosophers, who are not Mathematicians, it will not be wrong to say that they still regard the problem as unsolved and continually new solutions are being offered, as will be clear from the sequel.

Lee has discussed textual interpretations of Zeno's arguments at length[31] : I will however discuss them in the form in which they are most widely discussed and are

philosophically valuable, without entering into the question of Historical accuracy.

The first argument, known as the dichotomy is the following :—

"There is no motion, for what moves must reach the middle of its course before it reaches the end"[48·4] (p. 348*).

More explicitly stated, this means that before covering the whole distance a moving body must cover half the distance, and before covering half the distance, it must cover one fourth and so on ad infinitum. Since the process cannot come to an end, a body cannot reach the end of its course.

Russell remarks here that the infinite regress is not of the vicious type and no fallacy is involved. The vicious type of regress requires that a proposition cannot acquire *meaning* without the assertion of an infinite number of propositions. "Take for example the following : Two people are said to have the *same* idea when they have ideas which are similar; and ideas are similar when they contain an identical part. If an idea may have a part which is not an idea, such a definition is not logically objectionable, but if part of an idea is an idea, then, in the second place where identity of ideas occurs, the definition must be substituted and so on. Thus whenever the meaning of a proposition is in question, an infinite regress is objectionable, since we never reach a proposition which has a definite meaning"[48.4] (p. 349).

But the infinite regress involved in the statement "0 moves from A to B" is not of this type. We do not have to *mention* the motions of 0 on all the parts of the distance AB, in order to define the *meaning* of the proposition.

I doubt whether the difference between the vicious and the admissible infinite regress is sufficient to meet Zeno's challenge. The challenge does not lie in demanding a *meaning*

*The words in 2 are ..... "that which is in locomotion must arrive at the half-way stage before it arrives at the goal" (p. 335).

of the proposition "0 moves from A to B". It can be maintained that the proposition may have meaning, but it has contradictory implications, and therefore it must be regarded as false.

There are various views as to what these contradictory implications are. Aristotle and many others[37] have suggested that the contradiction lies in supposing that (1) the path AB has an infinite number of parts, but (2) the time required for covering AB does not have a corresponding infinite number of parts. If this is all, the contradiction can be easily resolved by pointing out that for every part of the distance AB, there is a corresponding part of the time required to cover AB.[2] (p. 320).

It has also been frequently suggested that the contradiction lies in supposing that (1) an infinite number of distances must together make an infinite distance, and on the other hand (2) the distance AB is finite.

This again can be easily disposed of by the principle that the sum of an infinite number of terms can be finite. The sum of all the part distances 1/2, 1/4, 1/8, etc., is one and not infinite as can be seen from any book on elementary Algebra.

The sting of the argument to my mind lies in the following :—

(1) The distance AB is made up of an infinite number of parts.

(2) 0 must cover these parts one by one.

(3) But it is not possible to cover an infinite number of parts one by one.

The last statement can be more precisely stated. As Ryle has suggested, suppose 0 decides to mark the points where 1/2th, 1/4th, etc., of the course ends. The point B will never be marked by this procedure, because it is a point where none of the course is left over, whereas 0's procedure can only give those points after which some portion of the course is left over. The inexorable injunction of the procedure is that 0

must never take the whole of the remainder. He must always divide it. The point *B* therefore is by definition excluded.

If 0 assigns numbers 1, 2, 3,.....etc., to the points marked, every point in the series generated by dichotomy will bear some finite number, and no point will be the last. This is what is meant by saying that 0 cannot cover an infinite number of points one by one.

But in another sense, 0 *can* cover an infinite number of points one by one. 0 may be at *A* at one definite time and at *B* at another definite time and at every point 1/2th, 1/4th, etc., at some definite time in between. If 0 insists on assigning numbers to these points, the point *A* will bear the number 1, 1/2th point will bear the number 2 and so on. Every point except *B* will bear some finite number, and *B* will come *after* all these points.

In the previous procedure *B* was excluded by definition. This was quite unwarranted. As Broad has pointed out, though we are right in saying that the points 1/2th, 1/4th, etc., are there on 0's path, we are completely wrong in assuming that there is no other points on his path. It is true that 0 comes to the point 1/4th after coming to 1/2th etc., and that each of these points is defined by successively halving the distance, but it is also true, that 0 comes to *B* after coming to all these points. We first deliberately exclude the point *B* from consideration, by deciding to consider only the points given by successive halving and then complain that we do not find *B* anywhere. We are in the same boat with the man who murdered his parents and then threw himself on the mercy of the court as an orphan.

If we are fishing with a net having holes of a particular size, we shall be able to catch only those fish which are bigger than the holes. This does not entitle us to say that the smaller fish are not there in the river. Similarly, 0's movement from *A* to *B* consists of certain points and the instants at which 0 is at these points. By confining ourselves to the successive halves,

we are using a net which will leave out many points and instants, the smaller fish. We then complain that the fish, *i. e.* the point *B* which ought to be there is not there, because we have not caught it in our net and this is a contradiction.

Bergson in one of his passages comes very near this principle, but very soon misses it. "We may disarticulate as we will the movement.....thus reconstructing....it.... in an arbitrary way, according to a law of ours, which may be incompatible with the real conditions of mobility"[4.1] (p. 252). The wrong phrase in this statement is 'thus reconstructing'. If we really reconstruct motion, our law of reconstruction cannot be said to be "incompatible with the real conditions of mobility". The fact is that division into successive halves is incapable of "reconstructing" motion, because motion has other parts besides those that are given by this process.

The fallacy in the statement, 0 cannot cover an infinite number of points, becomes obvious if we interpret it to mean "0 can *never* cover an infinite number of points". Thus interpreted, it amounts to saying that there is no time at which 0 is at the last point of its course, *viz.*- *B*. This is clearly false. There is a perfect correspondence between times and positions in 0's journey from *A* to *B*, and no contradiction can be demonstrated in this correspondence.

The Special Difficulties of Time :

William James and many other writers, have argued in this connection that the impossibility of completing an infinite series cannot be dissolved by demonstrating a correspondence between two infinite series. Even if we consider time alone and leave 0's motion from *A* to *B* out of account, the impossibility stares us in the face. An hour cannot elapse because before an hour elapses, half an hour must elapse and so on..... We come back to the same difficulty. No correspondence with another series is going to help us here. The pure time series involves the same difficulty[24.1] (pp. 229, 239).

James clearly overlooks the fact that the phrase "an hour has elapsed" is not possible apart from correspondence. Suppose some one has arrived late at a meeting and wants to know how late he is, he may be told "An hour has elapsed since the meeting began". The reply clearly means that the event of the beginning of the meeting is earlier than the event of the gentleman's arrival by an hour. Thus a proposition involving past tense cannot be made unless there is some event simultaneous with the proposition. In this case, the event in question is the gentleman's arrival. In a single time-series, no two events are simultaneous and therefore the proposition "an hour has elapsed" is impossible in connection with such a series.

In order to realize this, we have only to consider how Time is measured. As pointed out by Cantor, we measure Time by motion and not vice versa[9·2] (Vol. XXII, Sep. 1915, No. 7).

Some writers see a circularity here to the effect that ".....time is measured by means of motion; but motion presupposes the notion of time"[13] (p. 8). One may as well argue that "we measure lengths by means of a rod, but the rod presupposes the notion of length. This involves circularity." We are able to measure lengths by means of a rod, precisely because the rod has length. Similarly we are able to measure time by motion, because motion takes time. When all the corresponding positions of the hands of two clocks are simultaneous, (assuming that we can determine simultaneity), we regard the motions of the two clocks as equivalent and can take such equivalent motions — or clocks — as measures of time, just as we take rods of equal length as measures of length.

Thus the measurement of time is bound up with correlation. If there were a single clock, and no other motion (or change) in the world, the statement "an hour has elapsed" would have no meaning. An hour, by definition, is a class of equivalent motions, and with a single clock there is no question of equivalence (except in a reflexive sense).

The point will become obvious if we take a spatial measure. The sentence "this piece of cloth is a yard long" has meaning. It means that a rod called a yard and the piece are equivalent in length. But if the rod were the only thing given, and there were nothing else equivalent to it, it would not be meaningful to say that even the rod is a yard long. All we can say is that the rod has length, but how much that length is depends on the relation it bears to the lengths of other objects, *i. e.* on whether it is equal to shorter, or longer than, other objects.

It may be said that measurement of time and space may be bound up with correlation, but time and space themselves are not so bound up.

The answer to this is that in practice and in logical discussions, it is difficult if not impossible to talk of pure time and avoid its measurable aspect. Zeno in his arguments talks of measurable time and not of pure time.

For example, in the dichotomy he argues that a moving object must traverse half the distance *before* it traverses the whole. It is assumed here that the time required for travelling a part of the distance is *less* than the time required for travelling the whole distance. In the Achilles, it is assumed that the time required by Achilles to come to the place where the tortoise was, is equal to the time taken by the tortoise to come to where it is from where it was. In the arrow, the distance travelled by the missile is assumed to be composed of the distances occupied by the arrow, otherwise it cannot be assumed that the arrow can cover a distance, simply by occupying parts of that distance at every instant. The stadium clearly talks of equality of times. More, less and equal are clearly metrical notions.

It may be argued that the proposition 'an hour has elapsed since the meeting began', refers to nothing but the proposition itself and the beginning of the meeting and states that the beginning is earlier than the proposition by an hour. The proposition does not refer to anything simultaneous. Thus

only a pure time series is involved and no correspondence is required, to give meaning to the proposition "an hour has elapsed".

This argument is based on the view that a proposition about a proposition *P* can be identical with the proposition *P*. If it were so, it would be possible to say "what I am saying now is false". This is the famous liar's paradox. The paradox is based on the assignment of two incompatible meanings to the phrase "What I am saying now". In one sense the phrase stands for the whole proposition of which it is a part, in another sense it stands for itself. It is only by recourse to this incompatibility that the paradox is regarded both as the proposition *P* and a proposition about *P*. We can avoid the paradox by disallowing such a possibility. A proposition therefore must refer to something other than itself, and in the case of the proposition "an hour has elapsed" it must be some event simultaneous with the proposition. It cannot mean that an event *A* is earlier by an hour, than the proposition itself, it must mean that *A* is earlier by an hour than another event *B* (which is simultaneous with the proposition).

It should be noted that the proposition *P*, does not *state* that *B* is simultaneous with *P*, otherwise a proposition about *P* will again have to be identical with *P*. It is necessary that there should *be* an event *B*, simultaneous with the proposition *P*. It is not necessary that the proposition *P* should *state* that *B* is simultaneous with itself.

When James talks of the moments "counting themselves out"[24.1] (p. 239), he is unconsciously thinking of two series (1) the series of instants in the duration and (2) the series of corresponding propositions that the given instant is the n^{th}. The proposition "an hour has elapsed" only means that "the given instant is the last" is the last proposition of this second series and is simultaneous with the last instant of the hour. His difficulty in being unable to make this last proposition is the same old difficulty of the series of finite integers having no last

term, and is to be solved on the same lines. The lapsing of an hour would involve a contradiction, if the last instant of the hour can be reached or numbered by the process of assigning numbers one by one to the previous instants and the number of these previous instants is yet to be regarded as infinite. For demonstrating this contradiction, we must show an instant immediately preceding the last instant and this cannot be done. Thus no contradiction can be demonstrated. In order to be able to talk of the present in time, we need a relation of simultaneity over and above the relations of earlier than and later than. An event is present in relation to an event *B*, (generally a proposition about the event *A*) with which it is simultaneous. If there is only a pure time series, no event is present to any proposition and the proposition "an hour has elapsed 'now'" cannot be made.

It might be said that this involves an infinite regress. If we need a proposition simultaneous with an event, to enable the event to be present, we need another proposition to enable the first proposition to be present and so on. No such regress, however, is involved. The relation of simultaneity is symmetrical and if an event is simultaneous with a proposition, the proposition is equally simultaneous with the event, and no second proposition is required.

A reader might suspect that we are guilty of circularity here, if not of a regress. We are defining the presentness of an event by means of a proposition, and the presentness of the proposition by means of the presentness of the event.

Even this suspicion is unfounded. Circularity as a fallacy occurs only when it defeats its own purpose. If I say Madwi means Khadwi and after being asked what is meant by Khadwi, I reply "it means Madwi", I am guilty of circularity, because my purpose in giving a synonym to Madwi is to explain the meaning of Madwi. This is defeated by giving a synonym which is as unintelligible as Madwi. But if I say, *S* is not the only son because he has a brother *R*, and *R* is not the only son because

he has a brother, *viz.*- *S*, there is no circularity. There is no question of assuming what I have to prove here. This in not an argument, but a definition. *S* is not the only son if he has a brother *R* and *R* is not the only son if he has a brother *S*. Similarly, the event *A* is present in relation to an event *B* which is simultaneous with it and the event *B* is simultaneous in relation to an event *A* which is simultaneous with it.

Both the vicious infinite regress and the fallacious circularity involve a kind of contradiction. The infinite regress involved in postulating a serpent to support the earth arises out of the contradiction that the earth needs a support, but the serpent which in all relevant respects is like the earth does not need a support. A circular definition involves the contradiction that the same term needs a definition when it occurs as a subject but not when it occurs as a predicate. No such contradiction can be shown in our definition of the present.

Ryle has given an example of a cake. If every time only half of the remaining cake is is to be eaten, the whole cake can never be eaten, but this can only be attributed to our way of eating the cake. There is no logical impossibility in eating the whole cake[49] (p. 46).

Some people are apt to think that this example is not applicable to completing the whole course *AB* by motion, because the cake was made as a whole. The mother did not first make half the cake, then one fourth of it and so on and in the end add all these parts to make the whole cake. But *O* in moving from *A* to *B* does move half the distance before moving the whole and so on. In the case of the cake there is no need to complete an infinite process, whereas in the case of motion, this is exactly what must be done.

This seeming difference between the infinite parts of the cake and the infinite parts of motion arises from the fact that counting, like motion takes time. But this fact is not relevant to the uncountability (not to be confused with

non-denumerability) of the infinite. An infinite number cannot be reached by counting not because there is not enough time at our disposal to count upto it but because it is *not* one of the numbers that can be counted. We cannot find oil in sand not because we do not have enough patience to try till we find it, but simply because it is *not* there. Countability of the infinite is thus a logical and not a temporal impossibility. The cases of the cake and the motion are thus essentially similar, time being irrelevant to the problem.

When James contends that counting is the only way in which a moving body can cover the part distances, he is confusing two statements (1) counting is a process in Time and (2) Time is a process of counting. The two statements are clearly different. In counting there is a first and a last term and every term except the last has an immediate successor and every term except the first has an immediate predecessor. Even in a finite stretch of time, however, say one hour, there is a first instant and a last instant, but no instant has an immediate successor and no instant has an immediate predecessor (unless we hold a theory of Time which postulates this). The process by which a moving body moves from the first point in its path at the first instant and reaches the last point at the last instant is not similar to the process in which a passenger in a moving train counts the first telegraph pole, at a particular instant and gradually comes to count the 100th pole at a subsequent instant. If we consider these 100 instants only, they no doubt fulfil the conditions for counting. There is a first and a last instant and every instant except the last has an immediate successor, and every instant except the first, has an immediate predecessor. But these instants by themselves do not constitute time. They are only some instants chosen from Time. Time consists of many other instants besides these and their interrelations are not the same as the interrelations of these selected instants. Counting thus is not only the only way in which a moving body can cover the part distances in its path, but on the contrary it is a sure way of preventing the body from moving at all.

Upshot :

Zeno's argument known as the dichotomy rests on supposed contradictions in the notion of infinity. But such contradictions cannot be demonstrated in the case of motion. The main source of the difficulty is a confusion between two propositions, *viz.*- (1) Counting is a process in Time and (2) Time is a process of counting.

XII

The Reality of Motion

(2) Achilles and the Tortoise.

The second argument of Zeno is the famous race between Achilles and the tortoise. It runs as follows :—

"The slower will never be overtaken by the swifter, pursuer must first reach the point whence the fugitive is departed, so that the slower must always necessarily remain ahead"[48.4] (p. 350).*

The Achilles argument in essential respects is the same as the first argument. The distance between Achilles and the tortoise is diminishing, like the distance between the moving body and its destination, and since it diminishes in an infinite number of steps it is supposed that the process of diminishing has no end. In the case of Achilles. his destination, *viz.*- the tortoise is also moving.

Some writers think that this is of vital importance and if somehow the tortoise is prevented from moving the problem

*The words in 2 are "..... in a race the quickest runner can never overtake the slowest, since the pursuer must first reach the point where the pursued started, so that the slower must always hold a lead" (p. 335).

will be easier. The following plan is therefore suggested :—

"Suppose..... the tortoise is going at the rate of one inch per second and that Achilles is running after it at 500 inches per second, but suppose in addition that the race is taking place towards the stern of a boat drifting forwards along a river at one inch per second, then to an observer on a bank of the river, the tortoise will appear to be at rest and Achilles will be running towards it at 499 inches per second. Nothing now can prevent Achilles from reaching the tortoise ..... and the paradox vanishes"[60] (p. 61).

Surely, if the paradox could vanish by making the tortoise stationary, the first argument of Zeno would present no difficulty. The problem arises not from the motion of the poor tortoise, but from the fact that the distance between him and Achilles — exactly like the distance in the first argument — is infinitely divisible.

There is one point, however, which the Achilles brings out more clearly than the dichotomy. Aristotle has "solved" the dichotomy by pointing out that the time required for covering *AB* is capable of as many bisections as the space *AB*. In the Achilles both the time and the space are divided. The time required by Achilles to reach the place where the tortoise was is first considered and then the distance through which the tortoise will move ahead in this time is found out, at each step. Thus it is shown that though the time and the space can both be divided infinitely, this in no way serves to get over the difficulties of motion.

Various solutions to the Achilles have been offered. Wisdom in an article in the Mind observes :

"When Achilles starts from what to all outward appearances is rest, there will be a gap between our measurements that show he is at rest and those that show the first measurable velocity"[60] (p. 70).

In other words, if *A* is the whole distance which Achilles travels, he must travel some portion of it, however minute, to

begin his movement. If there is no such minimum portion, he cannot even begin to move.

We cannot therefore regard the distance as infinitely divisible. In any case, there is no proof that distance and duration can be divided beyond a certain finite limit. The distances and durations that we know, cannot be divided beyond a minimum and "of intervals and such like entities we might say that their esse is percipi"[60] (p. 73). Achilles is said to overtake the tortoise when the distance between them reaches this minimum. When this happens "Achilles..... would be abreast of the tortoise, so far as we could tell from our instruments"[60] (p. 67), which cannot measure beyond this minimum.

Such atomistic views of space and time have been held by others besides Wisdom, and are defensible.

Wisdom further observes that "we may interpret the dichotomy as being an attack on the concept of absolute rest ....."[60] (p. 69). In order to start moving, an object "has to begin at the far end of an infinite series"[60] (p. 67).

This argument appears valid only on the supposition that an infinite series has no end or a last term. But this is by no means the case. The series of points on a line *AB* of finite length is infinite but it has a first point, *viz.- A* and a last point *viz.- B*. The series of instants from 10 O' clock to 11 O' clock is infinite, but it has a beginning and an end. Again Wisdom does not seem to realize that, if this argument were valid it will not do merely to sacrifice "absolute rest". Motion has got to be sacrificed as well, because the argument shows that if a body moves from *A*, it can never reach any subsequent point because any such point is at the "far end of an infinite series".

Ushenko has suggested that Achilles does not have to travel over an infinite series of distances along the track in order to catch up with the tortoise. The track is a spatial abstraction. It is a series of events. The series is not the same when Achilles overtakes the tortoise, as when he started the race[55·1] (p. 161).

This may be so, but the difficulty (in Zeno's form) with regard to infinity does not disappear as long as it is not shown that the event of overtaking does not come at the end of an infinite number of events.

Ushenko has elsewhere offered a "final solution of Zeno's paradox of the Race". I must say it is very far from being "final" and his use of the theory of relativity in this connection, hardly seems relevant. He says...."When the world-lines of Achilles and of the tortoise representing their respective motions are drawn, they are bound to intersect, and the 'locus' of intersection is where Achilles would catch up with the tortoise"[55·2] (p. 242). Surely it was not necessary to bring in the "theory of relativity" in order to say that the curves standing for the motion of Achilles, and the tortoise intersect. The problem arises precisely because they intersect, whereas according to Zeno's argument they ought not to. If one argument proves the proposition not – P, it cannot be refuted simply by producing another argument which proves P. The task in such a case is to find out which of the arguments is fallacious. Ushenko has not even tried to show, in what way Zeno's argument is fallacious.

Ushenko seems to think that the substitution of space-time for space and time does the trick. The idea, perhaps is that Time involves the difficulty of completing an infinite series, and therefore if we dispense with Time by substituting the "Interval" *i. e.* the compound space-time for it, the difficulty disappears. But this is like supposing that the difficulties of procuring flour will disappear if we use bread in place of flour. Bread cannot be prepared without flour, and the interval cannot be defined without having recourse to space and time. ....."Suppose that an observer, treating himself as motionless, judges the distance between two events to be r, and the lapse of time between them to be t. Then if C is the velocity of light, the square of the interval is

$$c^2 t^2 - r^2 \text{ (or } r^2 - c^2 t^2)"^{48·2} \text{ (p. 307).}$$

Thus Ushenko will not succeed in eschewing time by taking recourse to space-time.

Again Ushenko forgets that there is no ambiguity about the time order of events on a given body and it is perfectly legitimate to consider the times at which Achilles and the tortoise occupy their respective positions. Both are moving on this earth, and we are also watching their race standing on the earth. "My space and time, as known in perception are correlated with those that, in Physics, are appropriate to axes that move with my body. Relatively to axes tied to a given piece of matter, the old separation of space and time still holds; it is only when we compare two sets of axes in rapid relative motion that the problems arise which the theory of relativity solves"[48·2] (p. 309).

Ushenko thinks that describing the race from the point of view of a terrestrial observer "....in terms of separate space and time, would give a private perspective, according to Zeno, an illusion"[55·2] (p. 242). Whether Zeno would have regarded a private perspective as "illusion" is a purely historical question, and Ushenko has not substantiated the ascription of this view to Zeno.

It is important to note in this connection that the problem set by Zeno to the world is that of describing motion in a way which does not involve contradictions. This is quite different from the problem of giving a "public" as against a "private" perspective of motion. Perhaps this is what Winn means when he says that Zeno's "is a mathematical and not a physical problem"[59] (p. 400).

King, like Ushenko and Carr [10·2] brings in relativity to bear against Zeno. He proposes to solve the difficulties of motion by reducing motion to rest. His words are, "..... all motion is relative, and by a proper choice of frames of reference, any specific motion may be 'reduced' to a mere endurance"[29] (p. 660).

It is not clear what King means by "any specific motion". Does he propose to reduce the motion of Achilles *in relation to the tortoise* to rest? As long as he does not show how this can be done by at the same time providing for the empirical fact that Achilles does overtake the tortoise, his contention can hardly be said to be relevant to Zeno's paradox.

If on the other hand he only draws attention to the principle that both Achilles and the tortoise can be said to be at rest in relation to some frames of reference, he does not seem to have contributed anything towards answering Zeno. It is difficult to see how the paradoxes that arise from *A's* movement in relation to *T*, can disappear even if *A* is stationary in relation to *C*, and *T* is stationary in relation to *D*. A lover carried farther every moment from his beloved in a plane can hardly be consoled on being told that he is stationary in relation to the plane, and she is stationary in relation to the earth, and thus both are at rest!

Russell solves the difficulty thus :—

Suppose the tortoise has started from a distance two furlongs and Achilles from a distance one furlong from the beginning of the race course. Suppose the race is over in one hour, and the speed of Achilles is twice that of the tortoise. Then if X stands for the portion of one hour that has elapsed since the beginning of the race, the position of Achilles at any time is given by the expression $1 + 2x$ and that of the tortoise by $2 + x$. When one hour is over $x = 1$ and both the expressions are equal to 3. Thus both Achilles and the tortoise are three furlongs from the beginning of the race-course and Achilles overtakes the tortoise. For any other time, even slightly less than one hour the two expressions give two different positions of Achilles and the tortoise[48·4] (p. 350).

In this race, the course of Achilles includes that of the tortoise, and still there are as many positions in the tortoise's course as there are in Achilles's. Some people are puzzled by this. It is supposed that since Achilles runs longer, he must

occupy more positions. But this is a mistake. Though the whole is greater than the part, it can have as many divisions as the part has. When a map is magnified, the original can be placed on a part of the magnified copy, and yet the magnified copy contains no additional regions, not contained in the small map. If it did, it would not be a copy but a new map. The notion of a magnified copy *ipso facto* implies the principle that two bodies can be exactly identical except in size.

Russell has used an analogous paradox for throwing light on the principle that whole and part can be similar. This is the Tristram Shandy paradox. Tristram Shandy is immortal. He gets a brain wave to write his autobiography. He took two years writing the history of the first two days of his life. In such a situation, commonsense would suppose that even if he does not weary of his task, he would never be able to finish his autobiography. But Russell argues that since every day lived has a corresponding year in which it is written about "no part of the biography will remain unwritten"[48·4] (p. 358 ff).

Morris says that referring to all the years in the life of an immortal person like Tristram Shandy, "involves an illegitimate totality, since Russell has expressly said that the series of days and years has no last term"[39] (p. 453).

This objection cannot be entertained because even if a series has no last term we can talk of all the terms in the series. The series of the numbers less than one has no last term, but we can legitimately talk of all the numbers less than one.

There is however some truth in saying that we cannot make true statements about "all the years in the life of an immortal person". Any such actual statement will be made at some time and will involve an assertion about events that have to occur after this time. Unless it is accepted that we can make true statements about the future and consequently that the future is real in some sense, there can be no true statement about these events.

But the relevant point in the Tristram Shandy is not whether a true proposition like "X is immortal" is possible. The paradox is designed to illustrate that (1) "two variables which start from a common term, and proceed in the same direction, but diverge more and more, may yet determine the same limiting class"[48.4] (p. 359). This conclusion does not conflict with anything in experience. The Achilles paradox is based on the opposite assumption that (2) if the path of Achilles is to be a whole which contains the path of the tortoise, Achilles must occupy more positions than the tortoise does. This assumption conflicts with the fact that Achilles *does* overtake the tortoise. We must therefore reject 2 and may accept 1.

Just as in the first argument successively halving the distance was a way of choosing some of the points on the path of a moving body, in the Achilles argument finding the corresponding positions of Achilles and the tortoise is a way of choosing some of the points on the path of the race. The number of these points is infinite, but they do not exhaust all the points on the path. The point where Achilles meets the tortoise is beyond these points. The fallacy just as in the first argument lies in supposing that, because this way of choosing the points gives us some points that are there on the path, it is capable of giving us all the points.

Morris states Russell's solution of the Achilles thus.... ...."although the path of Achilles is only a part of the path of the tortoise, since the tortoise is always a point ahead, in an infinite series the part is similar to the whole, and the path of Achilles is similar to the path of the tortoise. By the aid of the infinite the point head start loses its sting, what is one point more when the points are infinite in number"[39] (p. 452).

This is a sheer misstatement. In the first place it is the path of the tortoise, that is the part of the path of Achilles and not vice versa as maintained by Morris. Whose path is the whole, and whose is the part, is not decided by who is

ahead, but by the fact that Achilles occupies all the points which the tortoise occupies, while the tortoise does not occupy the points lying between the start of Achilles and the start of the tortoise. The overtaking of the tortoise, consists in the fact that both the paths have a common end and not in that...."by the aid of the infinite the point head start loses its sting". Morris further says that a one — one, correlation between the paths of Achilles and the tortoise can be set up regardless of whether Achilles is behind, even with or beyond the tortoise[39] (p. 453). Certainly, but the path of the tortoise cannot be regarded as a part of the path of Achilles when Achilles is behind the tortoise, because Achilles's path does not contain some points covered by the tortoise. Mere one — one correlation is not enough for overtaking. For overtaking (1) the path of the pursued must form part of the path of the pursuer, and (2) both the paths must have the same end.

Morris says that the supposition that the number of points and instants in any finite interval is infinite, "is empirically inadequate, since it cannot be shown that any specific path or time interval is composed of an infiinte number of elements"[39] (p. 457). But he forgets that the supposition that the number of elements is finite is also empirically inadequate. The example quoted from James does not establish this empirical adequacy. ...."bottles and coffee pots empty themselves by a finite number of decrements, each of definite amount. Either a whole drop emerges or nothing emerges from the sprout"[39] (p. 459). This example proves nothing as long as James cannot deny that the front portion of the drop comes out earlier than the back portion, and when the drop falls to the bottom of the glass, the lowest portion of the drop touches the bottom earlier than the top portion. The same consideration applies to the steps of Achilles which according to Bergson and Morris must be regarded as indivisible. Even within a step, the foot of Achilles does move from and to a place, it is at the earlier places at earlier times, and at later places at later times. The indivisibility of the

steps of Achilles can certainly not be justified on empirical grounds.

Those who, like Morris and Bergson take to a finitist account of motion finding that the notions of infinity and continuity are odd to commonsense, do not sufficiently realize that the finitist account is no less odd. Morris maintains that ....."a solution is possible if time and space do consist of a finite number of elements, provided that points are so defined that the points in the path of Achilles are not the same as the points in the path of the tortoise, in which case Achilles does not have to appear at the same points at which the tortoise appears"[39] (p. 457).

The proviso, that Achilles need not occupy all the points occupied by the tortoise is not necessary, if we suppose that the tortoise remains at a point for more than one instant while Achilles does not, thus explaining the difference in their speeds. But this, brings in, with a vengeance, the notion that a motion is a series of stops for avoiding which the Bergsonians take the finitist position. If on the other hand, nobody is at a point for more than an instant, Achilles will have to skip some points, in order to go faster than the tortoise, which seems to be the view adopted by Morris. But on this view, it will have to be supposed that if Achilles increases his speed beyond a certain limit he will either be behind or ahead of the tortoise, but never at the same place and thus never be able to overtake him. For example let the following figure represent the race.

	i_1	i_2	i_3			
	t_1	t_2	t_3			
p_1	p_2	p_3	p_4	p_5	p_6	p_7
A_1			A_2			A_3
i_1			i_2			i_3

P_1, p_2 etc. are the consecutive points in a discrete space, t_1, t_2 etc. are the successive positions of the tortoise at the consecutive instants i_1, i_2 etc. and A_1, A_2 similarly the successive

positions of Achilles. Achilles is so fast that he skips two points in his run from one instant to the next. But the curious result of being so fast is that he will never overtake the slowest creature; the tortoise. He must go to p_3 from p_1 in order to overtake the tortoise but this he can do only if he skips only one position, instead of two, *i. e.* if he slows down. This state of affairs is certainly not "empirically adequate". It shows how, if we allow our feelings of oddity, to govern the choice of our explanations, we may never be able to explain anything.

Aristotle seems to hint at the correct solution when he says "the tortoise is not overtaken, it is true while it holds a lead, but it is overtaken nevertheless if it is granted that it traverses the finite distance prescribed"[2] (p. 336). Thus in deliberately choosing any time before the race is over, for determining the respective positions of the runners, we are bound to find the tortoise ahead of Achilles. We cannot then complain that Achilles has not overtaken the tortoise. We search all our pockets for money but scrupulously omit the money bag, and then declare that no money can be found on our person.

Gerling here observes....."Whoever has still to prove the possibility of an overtaking is not yet permitted to speak of the time before or after which the overtaking takes place"[9,2] (Vol. XXII, June 15, No. 6).

The observation is not quite just. Even if the possibility of an event is in doubt, the time at which it can take place if at all, may not be in doubt, and in such a case we can talk of events before and after the controversial event.

Suppose in 2000 B. C., there lived a King who was worried about his queen not bearing a child. An ancient document says that he consulted his court-physicians, one of whom declared her to be barren, while the other disagreed with this diagnosis. We have no other information about the queen. In such a case, the possibility of the queen having

had a son is in doubt, but there is no doubt that if the queen had a son, he could not have lived two centuries before or two centuries after 2000 B. C. We can thus talk of events before and after an event, even if the possibility of that event is not established.

Jones, in an interesting article in the Mind has noted the crucial point by saying "Zeno considers the relative positions of *A* (Achilles) and *T* (the tortoise) over an infinite series of arbitrarily selected moments"[26] (p. 342). But he further ascribes to Zeno an assumption that by considering "each interval representing the time required to state that *A* (Achilles) moves to the point where *T* (tortoise) was and that *T* (tortoise) moves one half of that distance ahead", he is "considering positions at equal successive intervals of time"[26] (p. 344). This assumption is not at all necessary for Zeno. The crucial point is that the number of such intervals is infinite, and not that they are equal.

The Regulative and the Constitutive Infinite :

The conception of infinity is fraught with such difficulties that many philosophers feel, it could not be quite real, and some sort of secondary reality must be ascribed to it. As a result of this approach, a distinction between the potential and the actual or the regulative and the constitutive infinite, has been made. According to Kant, infinity is a regulative condition of our experience. "The infinite form of phenomena only means that we can go on perceiving, conceiving or imagining more and more about them, world without end. It does not mean that what we go on thus to represent shall have been there already by itself, apart from our acts of representation"[24.2] (p. 161). To take an example, a dog can go on chasing its own tail round and round indefinitely, and may imagine that the distance between him and the tail is infinite. In fact his act of chasing itself has created the distance. Any time he stops the chase, he will find that the distance he has turned round is finite, but since he can go on still further, one may say that infinity is the regulative principle of his movement (also see 46).

This way of looking at infinite divisibility implies that, the parts are created by our division and were not there before, just as the dog's act of chasing pushes the tail to a position where it was not. A stick two feet in length, does not contain a length of one foot until we divide it into halves!

A similar solution is in effect advanced by those who maintain that Zeno's arguments involve a confusion between "any" and "all". "Because Achilles has to go through every term of an infinite series, in order to reach the limit, it does not follow that he has to go through all the terms in order to reach the limit. The fallacy of the use of all in the collective and in the distributive sense ambiguously, is surely well enough known"[55.1] (p. 154). "The dynamic state of the flight allows for infinite divisibility in this sense : a suitably chosen obstacle can break the flight of the arrow at any point, but again not at all points in its course"[55.1] (p. 165).

All this amounts to saying that the arrow is at a particular place during its flight only if we choose to locate it, otherwise it is never there. The world of space and time is thus reduced to a mirror-image. We see our face in the mirror because we look into the mirror. We imagine that it will be there even if we do not look into the mirror.

For my own part I, feel that such "solutions" are evasions. I would prefer to say that the problem of infinity is not yet solved, and hope somebody will solve it in the future, rather than derive comfort from such "solutions".

Aristotle has explained his distinction between potential and actual infinity thus. "In the act of dividing the continuous distance into two halves one point is treated as two, since we make it a starting point and a finishing point, and this same result is also produced by the act of reckoning halves as well as by the act of dividing into halves"[2] (p. 383).

In the case of dividing into halves this is obvious. When a thread is broken into two, the end point of the first half, and the first point of the second half cannot be identical. But

in the case of reckoning halves, it is possible to suppose that the middle point on the line is the last point of the first half and the first point of the second half. We need not treat the point as two points.

Perhaps what Aristotle wants to point out is that even here the point is treated as having two sides, on one side of it lies the first half, and on the other the second. This amounts to regarding the point as two points.

This difficulty arises from not distinguishing terms and relations between terms. A point can have two relations without being two points. A player may be the best in one team, whereas in an overwhelmingly superior team he may be the worst. But in this case no body argues that the self-same player cannot be regarded as both the best and the worst, the player that is described in both these ways must be two players and not one!

Aristotle seems to imply that a distance does not really have an infinite number of parts, we impose infinite divisibility on it by regarding every point in it as two points.

But by carrying this reasoning to its logical conclusion, we have to suppose that the distance has no parts at all, let alone an infinite number of parts, because according to Aristotle, a line cannot be divided even into two halves without regarding one point as two points. A mile only potentially and not actually consists of two half-miles!

Upshot :

The Achilles argument seeks to show that even by regarding Time as infinitely divisible like space the difficulties of the dichotomy are not solved. The solution of the argument lies in the principle that the path of the tortoise is a part of the path of Achilles, though within any time during the race the tortoise covers some distance corresponding to the distance covered by Achilles.

XIII

The Reality of Motion

(3) The Arrow.

The third argument of Zeno is known as the arrow and is as follows :—

"If everything is in rest or in motion in a space equal to itself, and if what moves is always in the instant, the arrow in its flight is immovable"[48·4] (p. 350*).

Aristotle's answer to the arrow is that it is based on the assumption, that time is composed of indivisible moments. But "time is not composed of indivisible moments any more than any other magnitude is composed of indivisibles"[2] (p. 335). In any actual period of the arrow's flight, the arrow occupies a space, larger than itself and therefore Zeno's conclusion does not follow.

If Aristotle's solution is accepted whenever the arrow is in flight, it would have to be maintained that it "never" occupies a space equal to itself. It can do so only when it stops.

---

*The words in 2 are ".....if everything when it occupies an equal space is at rest and if that which is in locomotion is always occupying such a space at any moment, the flying arrow is therefore motionless" (p. 35).

Aristotle seems to admit that if there are instants, at any instant the arrow cannot be regarded as moving, and if it never moves, how it gets on, remains a problem. Zeno has immobilized the arrow in such a way that it casts "about in vain for means to pass to other stages"[18] (p. 50). Findlay in this connection has advanced an argument against Zeno. He points out that "even to be 'desperately immobilized' to 'cast about in vain for means to pass to other stages', would both, if they were anything, be states that lasted and took time. Our problem therefore takes for granted the very thing it finds so difficult"[18] (p. 50).

The argument seems to be *ad hominum.* In effect, Zeno is told that his arrow cannot "be" at rest because resting no less than moving takes time, and time involves all the difficulties of infinity and continuity. Zeno therefore cannot escape from them by fleeing from motion and taking refuge in rest.

This solution leads us nowhere. A dilectician like Nagarjuna or Shriharsh may argue on its basis that the arrow is neither at rest nor in motion and thus add to our difficulties. Regarding the arrow as stationary may be no solution, but the difficulty is not solved by saying this. Many others besides Findlay have charged Zeno of assuming motion while disproving it. But this charge can be levelled against any argument in the *reductio ad absurdum* form. In this form, one disproves a proposition by showing how the assumption of its truth leads to contradictions. Zeno is doing no more than this.

Heisenberg's principle of indeterminacy has been invoked for solving the arrow-paradox. It is argued that according to this principle, a particle in motion can be assigned a specific positional co-ordinate only at the expense of its specifiable dynamic properties of momentum and energy, "a moving particle is not at any definite position at any definite moment...."[55.1] (p. 162). Zeno is therefore wrong in assuming that the arrow is at a definite place at a given instant.

I do not think, indeterminacy is helpful in solving Zeno's difficulty. The principle in essence is that if we try to measure simultaneously two connected quantities like position and momentum, the more accurately we measure the one, the less accurately can we measure the other. The errors in both the measurements are systematically related. Their product can never be less than h/2.

This h is a very small quantity, and is relevant only to phenomena of microscopic minuteness. When we observe the sun, it is not affected by the fact that we are observing it. "But when the physicist tries to find out what is happening to an atom, the apparatus by means of which he makes his observations is likely to have an effect upon the atom." It has been found that "the sort of apparatus best suited for determining the position of an atom is likely to affect its velocity, while the sort of apparatus best suited for determining its velocity is likely to alter its position"[48.2] (p. 39).

Now Zeno's difficulty is not about microscopic phenomena, but is of a general nature, so as to apply to the movement of the sun and the moon as well. It poses a purely logical problem, *viz.*- whether the two statements (1) the arrow occupies different and definite positions at different instants and (2) it is moving throughout the period characterized by these instants, are contradictory. Pointing out that the first statement is false leaves the problem untouched, because nothing is said about the compatibility of the two statements. Logical paradoxes must have logical solutions.

Motion out of Stations :

According to Russell, the arrow-argument states the elementary fact that every value of a variable is a constant. The statement that the arrow moves is equivalent to the statement "X is a position of the arrow at an instant I", where both X and I are variables. True, the position of the arrow at an

instant is not its movement, but its different positions at different instants do constitute its movement. Though no single part of a table is the table, all the parts of the table properly integrated together are the table. What cannot be predicated of the parts separately may yet be predicable of the whole.

It may be said that the arrow at any instant is neither moving nor at rest. Movement implies that it should be at different places at different instants, while rest requires that it should be in the same place at at least two instants. Thus rest and motion are not exhaustive alternatives. On these definitions of rest and motion, it is correct to say that the arrow is not moving at any instant, but it is wrong to say that therefore it is at rest at that instant.

Some philosophers feel that there must be some difference between the arrow's position at an instant, when the arrow is moving as compared to its position at that instant when it is not moving. This difference is supposed by Leibniz to consist in the arrow having a definite velocity at every instant[48·1] (p. 83). Even when the velocity is constantly changing, we can calculate its velocity at any given instant.

But velocity is a purely external property. We can find it out only by knowing the positions of the arrow at different instants. If we only know the arrow's position at one instant alone, we cannot find out whether it has velocity or not. Thus velocity does not serve to differentiate the arrow's position at an instant, in flight and in rest.

King[29] has brought out the point that regarding motion as a series of states is a legitimate procedure by arguing on the following lines :—

A static space is an abstraction. What we really have is a configuration of the universe at an instant. The configuration at different instants may be different. Motion is simply the difference in the spatial relations within the configurations at different instants. It is therefore wrong to describe motion

as occurring *from* one point to another. The arrow whether it is moving or is static is a sequence of events, and to talk of a sequence of events changing or moving is like talking of Time taking time or motion moving. The sequence itself is Time, or movement.

It is clear that this view avoids the difficulties raised by such expressions as "the arrow moves" or "Time flows". But King is not altogether free from postulating a second time. He talks of the act of becoming being.... "*outside* the extensive continuum of space and time"[29] (p. 669). It is difficult to give a precise meaning to the word "outside". It is perfectly legitimate to ask whether becoming takes time or not. If it does, it is not outside time. If it does not and if Time is a series of becoming, it raises all the difficulties of the view that Time is a series of durationless instants.

Montague in his "The ways of knowing" has made a suggestion in this connection. "It is not the infinity of points or instants", he says "but rather the infinity of side by side relations or extension intervals between the points, and the infinity of lapses or succession intervals between the instants, which are the really significant and adequate constituents of distance and of duration"[38] (p. 180). If a foot and an inch are divided into successive halves, we get the following parts :—

1/2', 1/4', 1/8' .....
1/2", 1/4", 1/8", ...

Though the number of parts in both is the same, *viz.*- infinite, every part of the first is twelve times bigger than the corresponding part of the second. None of these parts are points, but may be regarded as distances between their end points. 1/2' for example may be regarded as a distance between the end point of a 1/2 foot long scale. They are therefore relations between points. Though even an infinite number of points cannot make a finite length, an infinite number of finite lengths can make a length equal to their sum.

Montague claims that this "conception of motion as a one to one correlation of elemental extension relations and succession relations (rather than points and instants) restores to motion its inalienable character of transition *from* one place *to* another; whereas the orthodox Mathematicians of our day oppose to our commonsense a Neo-Eleatic conception of motion as a mere succession of static occupancies of positions by bodies which themselves never really change *from* one position *to* another"[38] (pp. 180-181).

It is doubtful how far this claim is justified. Even if distances and intervals are regarded as more fundamental than points and instants, we cannot get rid of points and instants. If we try to do so, there will be nothing *between* which the distances and intervals lie. And as long as points and instants are there, the relations between them cannot preclude the possibility of a moving body being at a point at an instant, and there being no instant at which it was moving.

Jourdain and Grunbaum :

One of the important points in Russell's treatment of motion, in the course of resolving Zeno's paradoxes, is that of compactness. Compactness requires that if we take any two points on the path of a moving body, there must always be a third point between them, however near together we take these two points. Thus for any given point, there is no point immediately next.

According to Jourdain (and also Grunbaum[20] and Chari[11] after him), compactness is not enough for answering Zeno. If compactness is the only assumption about the continuity of the points on a trajectory, it can be shown that since every such point is contained in an interval as small as we please, the sum of these intervals is as small as we please. "Therefore the path of the arrow is as small as we wish. Therefore the arrow does not travel any distance"[27] (p. 53).

In order to avoid this difficulty, these writers suggest that the points on the path of a moving body must be assumed

to have a degree of continuity as high as that of the continuum of real numbers. Grunbaum further claims that by assigning this degree of continuity to a line, we can solve the time-honoured difficulty, as to how points without magnitude can form a line with a finite magnitude. His reasoning is as follows :—

In order to meet the argument that the points have no length and therefore even by summing together an infinity of them we shall never get a finite length, we must clearly understand what is meant by sum. In a sum, the terms that are summed are denumerable, *i. e.* they can be arranged in an order so as to give a point to point correspondence with natural numbers. When the terms form an infinite series, their sum is defined as the limit of the successive partial sums — the sums got by taking only some of them. We can now see that if the terms are non-denumerable, *i. e.* they cannot be arranged so as to give a point to point correspondence with natural numbers, we cannot speak of their sum. The points on a line are not only infinite but are non-denumerable, *i. e.* they are super-denumerably infinite. They therefore do not have any sum and the objection that the sum of the lengths of all the points on a line is Zero and therefore the length of the line must be zero, falls to the ground.[20·2]

I am extremely doubtful as to how far this line of approach is fruitful. Grunbaum has shown at most (if at all) that in the case of the super-denumerable infinity of points on a line, it cannot be *proved* that the sum of their lengths must be zero. But neither has Grunbaum proved that it *is* positive. For proving that a line is nothing more than an aggregate of points, such a proof is absolutely necessary. What is more, Cantor's ternary set which also consists of a super-denumerable infinity of points is not a line and as has been shown to have a measure zero[19] (p. 238). Super-denumerability alone, thus does not enable us to escape from the difficulty.

The truth is that length or distance is a relation between points, it is not a whole composed of points as parts. "The cat is 'in' the room" states a relation between the cat and the

room, *viz.*- the relation of inness. But "inness" is not made up by adding the room to the cat. The terms of a relation are not parts of the relation.

This of course does not mean that the terms are not parts of the complex formed by a relation and the terms. The cat and the room are certainly parts of the whole complex formed by the cat, the relation of inness and the room. What is denied is that the terms are the parts of the relation itself.

The Dilemma :

The arguments discussed so far are very widely known. But there is another argument of Zeno, which has attracted less attention though logically it is no less important. It runs as follows :—

"That which moves can neither move in the place where it is nor yet in the place where it is not"[9·2] (Vol XXII, Jan. 15, No. 1). Nothing therefore moves.

This form of argument is known as the dilemma. We shall therefore refer to it henceforward as the dilemma.

Nagarjuna the great Buddhist philosopher has advanced a similar argument in his Madhyamik Karikas. He says :

"A body cannot move on the path which it has traversed, and it cannot move on the path which it has not traversed, and there is no portion of the path apart from these two, where it could move*."

If the frontal tip of the arrow is at a point *P*, it has traversed all the points including and before *P* and all the points after *P* are such that it has not traversed them. If the point *P* is included in the points traversed, the two categories (1) points traversed by the arrow and (2) the points not traversed by the arrow exhaust all the points on the path of the arrow and therefore we cannot hope to find any portion of the path where the arrow can be said to move.

*गतं न गम्यते तावदगतं नैव गम्यते।
गतागतविनिर्मुतं गम्यमानं न गम्यते ॥[42] (p. 92)

If the point *P* is not included in the class of points traversed by the arrow, on the plea that any point traversed must be left behind some part of the arrow, the two alternatives do not exhaust all the points on the path, since *P* is not included in either of them. But the frontal tip of the arrow cannot be said to be moving on *P*. Movement consists in occupying two different positions at two different instants, whereas the tip of the arrow is at the point *P* only for an instant. Again we cannot get any portion of the path where the tip of the arrow could be said to be moving.

The past tense 'traversed' used by Nagarjuna, if strictly interpreted includes only those points on the path which are behind the tail of the arrow. The two categories (1) points traversed and (2) points untraversed therefore do not exhaust all the points. There is a third category, *viz.*- the points coincident with the arrow (*i. e.* points where it is). We therefore come back to the argument in Zeno's form.

The argument in order to be valid must therefore show that the arrow cannot move on the points in this third category, *i. e.* where it is. In order to do this, we need the help of the arrow argument, which says that the arrow occupies a space equal to itself wherever it is, and occupying a space equal to oneself is the same thing as rest.

The fifth argument thus reduces to the third.

An argument similar to the dilemma has been advanced to prove that a man cannot die. It runs as follows :—

"Men never die, for if a man dies, it must either be at a time when he is alive, or at a time when he is not alive; hence he never dies"[9·2] (Vol. XXII, Feb. 15, No. 2).

In the case of motion, we may argue :—

"A car cannot begin to move when it is moving, nor can it begin to move when it is not moving. A car therefore cannot begin to move."

In this case, it is clear that at the instant motion begins, the car is moving. We can therefore say that the car begins to move when it is moving. The statement appears odd, but perhaps it is less odd to say that when the car begins to move it is moving. The first statement follows from this. We can say either (1) I was bathing when he came or (2) when he came I was bathing.

Similarly, we can say that at the first instant of death, the man is not alive, therefore he dies when he is not alive. This sounds odd, but it is equivalent to the perfectly commonplace statement that when a man dies he is not alive.

Upshot :

That there is no instant at which an arrow can move does not imply that it is stationary at all instants, because both motion and station involve more than one instant. The objection that points cannot make a finite distance and instants cannot make a finite duration is an *ignoratio elenchi*, because distance and duration are relations between points and instants respectively and not wholes composed of them.

XIV

The Reality of Motion

(4) The Stadium.

The last argument of Zeno, which we shall now consider is known as the Stadium. It runs as follows :—

"Half the time may be equal to double the time. Let us suppose three rows of bodies, one of which (*A*) is at rest while the other two (*B*, *C*) are moving with equal velocity in opposite directions. By the time they are all in the same part of the course, *B* will have passed twice as many of the bodies in *C* as in *A*. Therefore the time which it takes to pass *C* is twice as long as the time it takes to pass *A*. But the time which *B* and *C* take to reach the position of *A* is the same. Therefore double the time is equal to the half"[48.3] (p. 180*).

*The words in 2 are "The fourth argument is that concerning the two rows of bodies, each row being composed of an equal number of bodies of equal size, passing each other on a race course as they proceed with equal velocity, in opposite directions, the one row originally occupying the space between the goal and the middle point of the course and the other that between the middle point and the starting post. This involves the conclusion that half a given time is equal to double that time" (p. 336).

Aristotle says that the fallacy in the reasoning lies in the assumption that a body occupies an equal time in passing with equal velocity a body that is in motion and a body of equal size that is at rest, which is false"[2] (p. 336).

Bergson has an explanation of how Zeno could have made such an obviously false assumption. "That, in the same time, a moving body passes different lengths of two bodies, of which one is at rest and the other in motion, is clear to him who makes of duration a kind of absolute, and places it either in consciousness or in something which partakes of consciousness. For while a *determined* portion of this absolute or conscious duration elapses, the same moving body will traverse, as it passes the two bodies, two spaces of which the one is the double of the other without our being able to conclude from this that a duration is double itself, since duration remains independent of both spaces. But Zeno's error, in all his reasoning is due to just this fact, that he leaves real duration on one side, and considers only its objective track in space. How then should the two lines traced by the same moving body not merit an equal consideration, qua measures of duration?"[4·1] (p. 253).

Bergson obviously thinks that consciousness is necessary for referring two "objective tracks of duration in space" to the same duration. It is difficult to see why. The fact that the two lines are traced by the same movement is enough to identify the durations, and thus to infer that the longer line was traced faster than the shorter. No consciousness is required. We take a stop-watch and note the positions of the runners A and B at the beginning and at the end of a minute. If the two positions of the runner B are farther apart from those of the runner A, we conclude that B is faster than A. We assume that the durations of the runs of both A and B are identical, because they were found to be congruent with the same duration, *viz.*- that of the movement of the hand of the stop-watch.

In fact Zeno is not likely to have committed the simple mistake as alleged by Aristotle. His argument is capable of a more justifiable interpretation. Tannery explains how the argument is directed against an adversary who maintains that "time" is a "sum of instants", but this does not mean that "each instant should apply to a fixed position of the arrow, but rather to the passage from each position to the next following position." As against this, Zeno "shows that the demand of his adversary cannot be granted, because it would make all motions equal". Thus—

> A motion from a point A to the next point on the left requires one instant.
>
> A motion from a point C to the next point on the right requires the same instant.
>
> Hence A moves relatively to C twice as fast as relatively to B.

It is therefore not the passage from one point to the next that corresponds to the instant, for it would then follow that the one equal to its double[9·2] (Vol. XXII Jan. 15, No. 1).

The argument obviously turns on two assumptions : (1) the instants are indivisible, and (2) they are consecutive. If the instants are not indivisible, it can be argued that if a motion requires the whole instant, a faster motion may require only a part of the instant. If the instants are not consecutive, it can be argued that if two bodies have passed the distance AB at an an instant t, the slower body may pass the next distance BC at an instant t', while the faster will pass that distance at an instant later than t but earlier than t'.

Thus if motion is described as taking place from one point at one instant to the next point at the next instant, any motion faster than this would have to skip some points, and any slower motion, would involve intervals of rest at some points.

It is possible that actual motion takes place in this discrete manner. Our instruments cannot detect times and

distances shorter than a certain minimum. If any points lying below this minimum are skipped, we shall never be able to notice it.

Identity of Substance and Continuity :

If a moving object 0 occupies a position p_1 at an instant t_1 and the next position p_2 at the next instant t_2, it cannot be urged as an objection that on this supposition 0 ceases to be the same object, and that the object at p_1 was different from the object at[10·1] p_2. (p. 66). We can talk of the emergence of a new object at p_2 only if the original object ceased to exist between t_1 and t_2. But in order to cease existing, there must be a time at which the object does not exist. Since between t_1 and t_2 there is no time, it is not possible to point out a time at which the object does not exist. The object at p_2 can therefore be regarded as the same as the object at p_1.

But there is a sense in which discontinuity of motion implies discontinuity of existence. This will be explained presently.

Two successive data, *viz.*- A_1 and A_2 are said to be the same object A, because they are similar and A_1 causally affects A_2. When this does not happen, we can always find a datum $A_{1\cdot5}$ between them which is more similar to A_1 than A_2 is and which bears the causal effects borne by A_1. If A_0 is a dog weighed just before 1 P. M., A_1 a dog exactly similar, and fed at 1 P. M., and A_2 a dog exactly similar weighed at 10 P. M., A_2 may not show a particular effect borne by A_1, *viz.*- weighing more by the weight of the food. We still think that the dogs A_2 and A_1 are identical beeause we believe that a dog $A_{1\cdot5}$ can always be found at 2 P. M., such that it is exactly similar to A_1 and A_2 and that it weighs more than A_0 by the weight of the food, *i. e.* it bears the causal effects borne by A_1. The moving object is said to be identical, because if we take a sufficiently short interval of time, the object at the end of the interval is closely similar and is causally affected by the object at the beginning of the interval.

It is clear that A_n does not bear all the causal effects of A_{n-m}, but only some. Some causal effects of A_{n-m} may be borne by B also, and still we do not say that A_{n-m} and B are aspects of the same object, bacause between A_{n-m} and B, we cannot find a continuous series of objects, such that objects nearer in the series, are more similar than the objects farther in the series, and bear closer causal relations, so that if we take an object near enough it bears almost all the causal effects of its preceding object.

If an object moves from one point at one instant to the next point at the next instant, we cannot have a series of similar objects between two adjacent points and instants. Thus theoretically, if the object at p_2 at t_2, does not bear some causal effects of the object at p_1 at t_1, there will be no way of determining whether 0_1 and 0_2 are two positions of the same object or not.

In this sense therefore discontinuous motion can be said to imply the ceasing of the old and the emergence of a new object from instant to instant.

In connection with the question whether motion is continuous or discontinuous, Whitehead's ideas have a special importance. He maintains that in analysing becoming into a subject and an act of becoming we are committed to the view that becoming takes place from one instant to the next. If A was A at one instant and B at the next, we can say that it is A that has become B. But if there is no instant immediately succeeding the instant at which A was A, how can we say that it is A that has become B? Before the emergence of B, there was a time when A was not there at all. If therefore A is to become B, there must be nothing between A and B. Thus the act of A becoming B is indivisible, it takes place at one stroke, and there is no earlier and later in it.

But A and B themselves are divisible both spatially and temporally. There are earlier and later parts of A and B.

Thus though becoming is indivisible and unanalysable, what becomes is divisible and analysable. "There is a becoming of continuity but no continuity of becoming"[58·1] (p. 53).

Applying these ideas to motion, the flight of an arrow, should be regarded as a series of indivisible acts, one immediately after the other. In this series the arrow before the act becomes the arrow after the act. But the subject of this becoming *i. e.* "the arrow before the act" is not indivisible or unanalysable. Thus though the act itself is indivisible, it is embedded in something which is analysable.

This is how, like Hegel, Whitehead tries to accommodate both continuity and discontinuity. He, in one way, concedes to Bergson the unanalysability of motion, but also makes room for it in the mover.

Whitehead's chief argument for regarding the act as indivisible and having an immediate successor, is that, if we don't do so, we cannot say "*what*" becomes"[58.1] (p. 106). I am not inclined to regard this argument as conclusive. We can talk of a subject of becoming even, if there are no immediate successors in the act of becoming. In fact, we are not entitled to say that A_1 has become A_2 even if A_1 is immediately followed by A_2. Like the Buddhists, we can maintain that A_1 and A_2 are different and it is not A_1 that has become A_2. As pointed out in analysing change, the subject of this becoming is neither A_1 nor A_2, but A, to which they both belong, *i. e.* both A_1 and A_2 *are* A[s]. In order that A_1 and A_2 should both belong to A, it is not necessary, that A_2 must be an immediate successor of A_1.

Upshot :

The stadium argument can be answered by assuming that space and time are continuous and motion does not take place from one point at one instant to the next point at the next. The assumption of continuity also serves to give a secure meaning to identity of substance.

XV

The Reality of Motion

(5) The Doctrine of Momentariness as applied to Motion.

N.B. :— The original arguments for the doctrine of momentariness are intended to apply to change in general. I have modified them here to deal specifically with motion.

The supposition that if there is motion, there must be something that moves has been utilized by the *Kshanabhangavad* school of Buddhism, in order to deny the reality of motion. According to this doctrine nothing lasts for more than a *Kshana* or a moment. It is therefore meaningless to say that anything moves. Motion implies that the same body occupies two positions at different times. But the same body cannot exist at different times, and therefore "there is no motion*" declares Vasubandhu. What appears as motion is a sequence of momentarily existing, different but closely similar objects, in different positions.

The doctrine is supported by the following arguments :—

The essence of existence is *Arthakriyakaritva.* Whatever exists has a definite function. Its existence in some way makes

*न गतिर्नाशात्, संस्कृतं क्षणिकं यतः।[56] (p. 85).

a difference to other existents besides itself. In other words it has causal efficiency.

If an existent is identified with its causal action, it becomes obvious, that no existent lasts for more than a moment, for no activity is identical at two moments. The causal action of a jar for example is different at any two moments. If one is continuously looking at a jar, there is at every moment a new perception of the jar. This is apparent from the fact that the continued presence of the jar is needed for its continuous perception. If the jar is removed it ceases to be perceived. (We do not subsume phenomena like after images under perception). Since any moment of the absence of the jar has a corresponding moment of its non-perception, and any moment of its presence has a corresponding moment of its perception, it is clear that at every moment the jar is producing a new effect, *viz.*- its perception.

The difference in causal action at different moments is even more obvious in the case of a moving object. If the jar, instead of being stationary is being carried by some one, at every moment, it is exerting its weight on a different part of the ground, it is casting its shadow on different places at different angles and so on.

Since the jar produces new effects at different moments, it is not proper to speak of an identical jar, if a jar is identified with its causal action. A jar that produces a new effect is a new jar and not the old self-identical jar persisting at different moments.

In fact, it is absurd to suppose that there is a stable object, which undergoes a series of causal acts. Let us take the example of motion. The stable or abiding object is 0, and the series of acts it accomplishes is occupying different positions, P_1, P_2, P_3. . . .etc. Now why is it that 0 cannot perform these acts all at once? If it is the same 0 which occupies P_1 as the one which occupies P_2, there is no reason why it can occupy P_2 only after it has occupied P_1.

The only reason can be, that at the time 0 occupies P_1, it is able to occupy P_1 only and it is not able to occupy P_2. We have therefore to postulate a new category, *viz.*- the ability of 0 to occupy a particular position. The abilities to occupy different positions are different and they are not all present in 0 at the same time.

Now if it is these abilities which enable 0 to execute the different acts, we are not justified in ascribing the acts of occupying different positions, to a persistent 0. The real cause is not 0, though the abilities which are the real causes seem to inhere in 0. 0 with the ability is different from 0 without the ability. Why should we then not straightaway regard the two as different objects and do away with the superfluous 0°. The only justification for calling both of them 0 is that there is a close similarity between them. But similarity is not identity.

Some people say that the so called abilities are not newly emerging objects. When 0 is associated* with A, it may be able to perform one act, while when it is associated with B it may be able to perform another. When 0 is associated with the position P_1, it may be able to perform the act of going to P_2, not otherwise, and so on. Thus the same 0 may be able to perform different acts, with different associates.

The Buddhist replies that the associates P_1, P_2 etc., must in some way affect the 0. If they don't, they serve no purpose and can be dispensed with. If 0 without the associate P_1, is the same as 0, with it, *i. e.* if 0, when not at P_1 is the same as when it *is* at P_1, it should be able to perform the required act — that of going to P_2, even without being at P_1.

If your 0 is not affected by the associates P_1, P_2 etc., it is like pure space which is not affected by anything such as heat and moisture. It cannot act in one way by becoming hot in association with heat, and in another by becoming wet in association with water. It cannot have any causal efficiency.

*क्रमवत् सहकारित्वभात् स्थायिनोऽतीतानागतयोः क्रमेण क्रमणमुपपद्यते ।[34] (p. 19)

In order to have causal efficiency and therefore existence, your 0 must be like leather. Unlike pure space, leather produces different effects in association with different associates. It becomes wet by contact with water and produces the sensation of warmth. If your 0 is like leather, it cannot be said to be identical when it is subjected to heat and to water. Dry and warm leather is not identical with wet and cold leather, however, similar they may be as leather.*

It must be pointed out here that the Buddhist is not committing the simple fallacy of supposing that because X and Y together have a property which X alone does not have, X by itself must be qualitatively different from the X in X and Y. 0, at P_2 has an ability to go to P_3, not 0 and P_2. It is 0 that moves, not 0 and P_2. Thus the ability to move must be possessed by 0 and not by 0 and P_2 together.

An objector says, "0 does not get new qualities and abilities, by being at different positions. The difference in the acts is due to the difference in the positions, and not due to difference in the 0. It is the very nature of 0 to act in association with the different positions, and the self-same nature can have different effects on different things.** A ray of light is reflected by a mirror, but absorbed by clay. The difference in action in the two cases is due to the difference in the mirror and the clay, and not due to any change in the ray itself. In fact, the very nature of the ray consists of all its effects on all possible objects. The same ray has all these effects, it is the objects that are different.

Similarly the ability to go to P_3, which 0 has when it is at P_2 but does not have when it is at P_1, is due to the difference in P_1 and P_2 and *not* due to any change in the nature of 0 itself."

*वर्षातपाभ्यां किं व्योम्नश्चर्मण्यस्ति तयोः फलम् ।
चर्मोपमश्चेत्सोऽनित्यः खतुल्यश्चेदसत्फलः ।।[34] (p. 20)

**भावस्तैः सहकारिभिः सहैव कार्यं करोतीति स्वभावः[34] (p. 20)

The Buddhist reply to this would be :—(1) If all possible effects of the ray constitute its very nature, whenever a ray is there, all its effects must be there. When only some effects are present, we are not entitled to say that the ray is present. This principle leads to the absurd result that whenever there is a ray, it must fall on all objects in heaven and earth, a ray cannot illumine only some objects.

If it is supposed that association with P_2 creates a new property in the abiding object 0, and in virtue of this property 0 can go to P_3, we land ourselves in an infinite regress, for the question at once arises. "What is the cause of this new property?" It was agreed that for producing any effect, a cause requires a suitable associate. This associate produces a new property in the cause, which in turn produces the effect. When the new property is regarded as an effect, its cause, whatever it be, must have a suitable associate which produces in it a new property No. 2 which produces the new property No. 1 and so on..... With this infinite regress, 0 will never be able to get on to P_3 from P_2.

All these difficulties are solved if we do not cling to the abiding 0, but suppose that the 'new properties' alone exist. Since these new proporties are defined by nothing else but the causal action which they actually perform, we do not need to postulate any lack of associates for explaining their inaction. The moment they are inactive they are non-existent, their moment of action is their moment of being. The Buddhists call this new property *Kurvadrupatva**.

Continuity and Discreteness :

Stcherbatsky has summed up the Buddhist position about the moment thus....."Ultimate reality for the Buddhist.... is a mathematical point-instant, the moment of an action's

*अथ भावादभिन्नोऽतिशयः सहकारिभिराधीयते इत्यभ्युपगम्यते तर्हि प्राचीनो भावोऽनतिशयात्मा निवृत्तः अन्यश्चातिशयात्मा कुर्वद्रूपादिपदवेदनीयो जायते इति फलितं ममाऽपि मनोरथद्रुमेण ।[34] (p. 22)

efficiency"....."the shortest time, the mathematical point-instant, is something real, since it is established by science.... the motion of a thing during a single moment they called instantaneous motion, or the motion of just that time, *i. e.* not of another time, not of another moment. This time is nothing but a differential of a planet's longitude[53]" (p. 106).

I do not wish to enter into the historical question as to how far the Buddhists were conversant with the ideas of the Calculus. But it must be pointed out that the greatest difficulty in the way of interpreting the instants of the *Kshanabhangavad* on the lines of the Calculus is that contrary to the assumptions necessary for the latter[48·4] (pp. 343-344), they seem to have been regarded by the *Kshanabhangawadins* as forming a discrete series.

This is clear from the statement that a phenomenon comes into being at one moment and is destoryed at the *second** and therefore it is momentary. The argument will follow only if the second means next, such that there are no moments in between the first and the second, at which the phenomenon could exist.

Continuity and Causation :

The commonsense notion of cause requires that the cause must immediately precede the effect. If there is anything between the cause and the effect, this something in between, if it invariably precedes, has a greater right to be called the cause. If there are an infinite number of variable occurrences between the cause and the effect, the ordinary notion of causaion is not applicable, because there is nothing which immediately precedes the effect.

On these considerations, we have to say that it is not the the seed that is the cause of the sprout, since between the seed and the sprout, there are other existents which invariably

*यत्रोत्पन्नस्तत्रैव द्वितीयक्षणे धर्मो विनश्यति ।[54] (p. 86)

precede the sprout. Instead of supposing therefore that a self-identical seed causes the sprout, we should suppose that there is a series of existents from the so called seed to the so called sprout and there is nothing permanent abiding in and through them all.

But this solution does not get rid of the infinite regress, as long as we cannot answer the question; what existent immediately preceded the sprout. If nothing immediately preceded the sprout, there must always be an existent after a given existent, but before the sprout, and so on ad infinitum.

This infinite regress is not of the vicious type, because it does not arise out of a postulate. An existent preceding the sprout is not a postulate, but a fact, and this existent can be there, whether there is anything between it and the sprout or not. On the other hand, the infinite regress arising out of the supposition that the seed is the cause of the sprout is vicious, because this supposition amounts to a postulate that the sprout immediately follows the seed, and when it is seen that it does not, an infinity of new properties are postulated to explain why it does not. An infinite regress that serves no other purpose but that of saving a postulate is vicious.

One of the arguments in favour of *Kshanabhangawad* is that destruction is natural and needs no external cause. "Destruction is negation, and negation needs no cause*" says Vasubandhu.

This principle can be argued out on the basis of the Buddhistic equation of existence with a causal act. It is the very nature of an act to bring about what is naught, and to annihilate what is. In the case of motion this is obvious. At any moment of its movement a moving body is annihilating its contact with the place where it is. If it is at P_1, it is in the act of leaving P_1. Thus the very nature of the act is annihilation of what is.

*न कस्यचिदहेतोः स्याध्देतुरेव विनाशकः[56] (p. 86)

In fact it will be wrong to say that a moving body is at a place, a more correct description would be that it is 'through' a place.

Prabhachandra's Objections :

Prabhachandra raises the following objections against *Kshanabhangawad.* He asks "does the momentary thing produce the effect, when it is destroyed, or when it is undestroyed*?" The first alternative is clearly absurd. Something which is no longer in existence cannot produce an effect. On the other hand, if it produces an effect when it is undestroyed, the momentariness is no longer there. The cause exists at least for two moments, one before the production of the effect and the other at which the effect is produced.

In the case of motion Prabhachandra might ask, does a moving object leave the place where it is at the time when it is there, or at the time when it is not? It cannot leave the place at the time it is there, unless it is there for two instants, one instant for staying there and another for starting to move from there. Thus, momentariness of the moving object's occupation of a position is sacrificed. The second alternative is obviously absurd. A thing cannot leave a place where it is not.

The *Kshanabhangawada* identifies an existent with a momentary causal act. It does not admit of a thing which acts. Prabhachandra's question is thus misdirected. The very nature of a causal act is to produce an effect. It follows therefore that the effect is produced when the causal act is undestroyed. This does not imply that the act lasts for more than one moment, it exists only at the moment the effect is produced, and not before.

It may be supposed that "leaving the place" is nothing but going to another place. If the moment of being at a place, and the moment of leaving that place are identical, a thing will be at two places at the same moment.

*क्षणिकं वस्तु विनष्टं सत्कार्यमुत्पादयति . . . . अविनष्टं वा ?[45] (p. 499)

The argument is based on an illegitimate identification of 'leaving a place' and being at another place. If an object moves from P_1 to P_2, it is at a place other than P_1 when it arrives at P_2, not when it leaves P_1. Going to another place is identical with arriving at that place and not with leaving the former place.

Prabhachandra further argues that the so called moment is not indivisible. It can have many effects, and in such a case it is not one moment but many. The sparkle of a flame*, burns the wick, as well as sucks the oil. The one causal act has many such effects. Where is then the oneness and indivisibility of the moment?

Prabhachandra ignores that, a succession of effects after a particular cause, cannot all be referred to that cause. That which precedes the effect more proximately has a greater right to be called its cause. One causal act, by definition cannot have successive effects.

On the other hand if the cause has many simultaneous effects, *Kshanabhangawad* is not affected. In fact, the movement of the scales is sometimes cited to show how one causal act can have two effects. One scale goes up at the very moment at which the other goes down. The same causal act, *viz.*- that of the greater weight — produces both these effects simultaneously. Similarly, the destruction of the cause and the production of the effect can be simultaneous**.

This principle, as applied to motion implies that the moments are discrete. Here the cause is the position of the moving body at P_1, the destruction of the cause is its separation from P_1 and the production of the effect is its arrival at P_2. Now arrival at P_2 and separation from P_1 can be regarded as simultaneous only if, the body can at no time be in between P_1 and P_2.

*प्रदीपादिक्षणाद् वर्तिकादाहतैलशोषादिविचित्रकार्याणि
शक्तिभेदनिमित्तकानि व्यवतिष्ठन्ते[45] . . . . (p. 503)

**नाशोत्पादौ समं यद्वन्नामोन्नामौ तुलान्तयोः ।[45] (p. 497)

Otherwise even after leaving P_1, the body may not be at P_2, but at $P_{1\cdot5}$ ahead of P_1, but still behind P_2.

Upshot :

The doctrine of momentariness has shown that the world of Time can be regarded as a series of momentary acts. But it has not adequately analysed the notion of an abiding substance and the necessity of continuity. Some texts indicate that the instants are to be derived on the lines of the Calculus, whereas others clearly regard them as discrete, and yet indivisible. Regarding the instants as durationless and yet discrete raises the time-honoured difficulty, as to how these durationless instants could make a finite duration. This question has not been faced by the *Kshanabhangawadins*.

XVI

The Reality of Motion

(6) Zeno's arguments on the whole and the hazards of infinity.

The arguments of Zeno on the whole, turn on two points (1) divisibility and (2) the logical difficulties involved in such a divisibility. The dichotomy and the Achilles obviously involve infinite divisibility. The arrow does not expressly require it. But in any finite portion of the time taken by the arrow's flight, the arrow occupies a distance greater than itself. Unless we divide the time into infinite parts, we do not get the instant without duration at which the arrow occupies a distance equal to itself and therefore must be regarded as at rest. The dilemma has already been shown to be essentially the same as the arrow. The stadium clearly assumes indivisible and consecutive instants, and shows that they involve their own difficulties.

The dichotomy, Achilles and the Stadium assume motion and then try to show the absurdity of the assumption. In the dichotomy, a moving object is said to reach the middle of its course *before* it *reaches* the end. In the Achilles, the faster is said to *reach* the place where the slower was, and in the stadium, the bodies are said to *pass*.

But in the arrow, the argument proceeds without assuming motion. "At all times the arrow occupies a space equal to itself, and this is nothing but rest" is the essence of the argument, and it does not assume motion at all.

Tannery has put Zeno's arguments in a connected form, so that they no longer appear independent but as a dialectical development of the same theme. The dichotomy shows the impossibility of continued bisection whereupon, the adversary, like Aristotle says that the bisection is not to be carried to an actual, but to a potential infinity, just as one may eat a cake in pieces of finite size, though the pieces could have been made smaller. To this, according to Tannery, "Zeno replies by stating the Achilles paradox, which does not involve bisection and in which the time interval is subdivided, much in the same way as the space interval"[9·1] (p. 18).

In the Achilles, it is not possible to talk of potential infinity. Achilles actually and not only potentially occupies all the positions occupied by the tortoise (unless on the basis of some novel theory we deny this). Tannery thus makes an important point in saying that the Achilles does not involve bisection and this is its special feature as compared with the dichotomy. But I think the second feature which he points out, *viz.*- that in the Achilles..... "the time interval is divided much in the same way as the space-interval" should be reserved for the 'Arrow'. Tannery himself says while introducing this argument. "The adversary" after being confronted with the Achilles — "takes the position that he has admitted too much. Finite time, he claims, is capable of division into an infinity of parts. Is there not a sum of instants? May there not correspond an instant to each successive position?"[9·1] (p. 18).

The arrow shows the difficulties of this position by showing that being at a place, at an instant, is rest and not motion.

The adversary then shifts his position and says that.... "he did not mean that each instant should apply to the fixed position of the arrow, but rather to the passage from each position to the next following position"[9·1] (p. 19).

Zeno assails this position by his stadium. If "....passage from each position to the next", always takes place in an instant, all motions must take place with equal speed and a body cannot cover double the number of positions, covered by another body, in the same time, without involving the absurd conclusion that half the time is equal to double the time.

It should be noticed that the dichotomy, the Achilles and the arrow do not need the assumption that the series of instants and positions is discrete. But the stadium does not follow without this assumption. If the number of instants and positions in a finite time and space is infinite, time cannot be measured merely by the number of positions occupied.

Zeno's arguments have proved a fruitful field for philosophical disputations in general, but they have a peculiar relevance to Hegelian philosophy. M. G. Noel maintains in the interests of Hegel that the first two arguments refute infinite divisibility while the next two refute indivisibles[48·3] (p. 173). Hegel says, "In both the first proofs, continuity and progression has the predominance, there is no absolute limit, but an overstepping of all limits." Here (in the third proof) "the opposite is established; absolute limitation, the interruption of continuity, without however passing into something else; while discretion is presupposed; continuity is maintained"[21] (p. 275).

Thus according to the Hegelian way of thinking the paradoxes of motion arise, because a partial truth is supposed to be the whole truth. Motion, space, etc. are both continuous and discrete. "If space were simply discrete, *i. e.* chopped up into discreteness, then it would be composed of indivisible units. If it were simply continuous, then it would be divisible ad infinitum, but neither of these is the whole truth"[52] (p. 158).

If we adopt this Hegelian way of thinking, we can "resolve" or rather swallow all contradictions in one gulp, but then we are also deprived of the soul of all reasoning, *viz.*- consistency. I am therefore suspicious of such facile methods. Moreover, the distinction between partial truth and whole truth does not seem to be quite relevant here. If one side of a cloth is black and the other white, we may say that regarding the whole cloth as black or white will give rise to a contradiction, since the property of a part is ascribed to the whole. But continuity is not the property of a part of space, and discreteness of another part of space. They both either belong or do not belong to space as a whole.

Stace suggests that if space is both continuous and discrete, it will not contain indivisible parts and yet not be divisible ad infinitum. This is just stating the contradiction and not solving it.

In recent times, Grunbaum has tried to answer Zeno. He takes over from Whitehead the conception of change as a series of events or acts which are indivisible and durationless. The subject of these acts however has duration and infinite divisibility. To take an example from James, the emptying of a kettle drop by drop is a series of acts. The coming out of a drop is an act. This act has no earlier and later portions. It is indivisible. But the drops themselves have spatial and temporal extension. The drop is not the same at any two instants and thus it can be analysed into a history.

Grunbaum however does not regard his acts as consecutive. Between any two of them a third can be found.

To the Zenoian objection that a series of such acts or events is infinite and interminable, Grunbaum answers that.... "the time required for the occurrence of that discrete denumerable sequence of events is *Zero* and therefore the existence of that sequence can hardly preclude the eventuation of the terminal event in the series"[20.1] (p. 180).

One would naturally comment that if a series of events takes Zero time to occur, it is as difficult to conceive its possibility as when it takes infinite time as alleged by some interpreters of Zeno.

The Web of Infinity :

We have seen that the dichotomy and the Achilles, explicitly turn on the difficulties of infinity. Modern Mathematicians have shown that infinity can be precisely defined and applied to all relevant cases without contradiction. The claim of James and others, that Time is a special case where such an application is not possible has been examined already. But there are a few other writers who still hold that infinity is still inscrutable. We shall now discuss the difficulties raised by some of these writers.

Max Black while considering the difficulties of infinity in connection with Zeno's paradoxes has brought in an interesting example. Hercules cut off the head of Hydra, but the moment it was cut off there was another head in its place. If this happens an infinite number of times, one can argue that Hydra had and did not have a head in the end. He had a head, because his head was replaced every time it was cut off, and he did not have a head because every head was cut off. The notion of infinity thus involves a contradiction[5] (p. 105).

This contradiction arises because Black has not specified what sort of infinite series is formed by the process of head-cutting. If after the first head was cut, another head took some finite time to grow, so that there was no head cutting between the first cutting, and the growing of the second head, and so on, so that each head cutting excepting the first is immediately preceded and followed by some head cutting, the series has no last term, if it is infinite and therefore the question whether Hydra had a head in the "end" does not arise, because there *is* no end.

But if the head-cuttings are like the successive occupations of points by a moving body, so that at any two times, however, near we take them, head-cuttings always occur, the series can have an end, just as a moving body occupies the last point at the last instant, in the end. In such a case, Hydra will have no head in the end, because when the number of cuts is equal to the number of heads, there is no head left.

The point to note here is that the internal order of a pair is not affected by the number of pairs. If a number of couples stand in a queue, such that the husband is always at the back of his wife, one can always say that the last person in the queue will be a man, whatever the number of couples in the queue. Similarly, in the case of Hercules and Hydra a head is always followed by its cut, and therefore the last occurrence *if there be any* in the series is a cut and not a head. If Hydra had no head to start with and a cut by Hercules's weapon caused a head to grow for the first time and so on ad infinitum, the last term in the series will be a head and Hydra will have a head in the end.

There is thus no contradiction involved in the conception of infinity, since it cannot be shown that *under identical conditions* Hydra will both have and not have a head.

Slenszynoki has raised the following difficulty about infinity — and consequently about Zeno's argument. "Thought is composed of a number of acts each of which requires a finite interval not less than a certain size. The number of these acts is therefore finite for every one. Consequently it is impossible to know an infinite number of moments"[12] (p. 194).

Such pseudo difficulties about infinity have been frequently raised in philosophy and James rightly makes fun of them. If any number can be known only by an equal number of acts, let alone infinity — it is even doubtful if we know any large number. I am sure that I know what is meant by saying that the population of India is 400000000. But I have not undergone 400000000 acts of thought in order to know

this. If "how can the finite know the infinite?", be a genuine difficulty, "how can the thin man know the fat?" can also be regarded a profound philosophical question.

Ranade remarks "The Pythagorean idea of discontinuity of space and time was abhorrent to both Zeno and Aristotle; they both regarded their infinite divisibility as a mere chimera"[47] (p. 51). This remark seems to imply that Aristotle regarded infinite divisibility, incompatible with continuity. But further on Ranade himself states Aristotle's definition of continuity thus : "A thing is continuous when of any two successive parts, the limits at which they touch are one and the same and are as the word implies, held together"[47] (p. 51).

Now it is difficult to see how continuity in this sense can be regarded as incompatible with infinite divisibility. The definition implies that whenever a continuous series is divided into two parts, such that there are no terms in the series which are not found in either of the parts, the last term of the first part must be the first term of the second part (assuming that the parts *have* first and last terms). This condition obviously excludes series like those of integers. The integers from 1 to 20, can be divided into two parts (1) those from 1 to 10 and (2) those from 11 to 20. These two parts contain all the integers that are contained in the whole series from 1 to 20. But still the last term (*viz.*- 10) of the first part, is not the first term (*viz.*- 11) of the second part. They are different. This is possible, because there are no integers between 10 and 11.

This shows that a series cannot be wholly contained in two of its parts, (which *have* first and last terms) without having a common term, if, whichever two of its terms you take, there are always terms of the series in between them. In such a case, if the last term of the first part is not identical with the first term of the second part, the last term of the first part and the first term of the second part, will have terms of the series in between them and thus the two parts will never contain all the

terms of the series. Thus continuity as conceived by Aristotle is not incompatible with infinite divisibility.

This also answers the charge of William James that the Mathematical notion of continuity is contrary to the commonsense notion, that "anything is continuous when its parts appear as immediate neighbours, with absolutely nothing in between"[24·2] (p.187). When a line is divided at a point P, such that P is both the last and the first point of the two parts respectively, there is nothing between the two parts. As long as the parts of a line are lines, the commonsense notion of continuity is easily applicable. The trouble starts when the points on a line are regarded as parts of the line, because we cannot say that there is 'nothing' (*i. e.* no points) between any two points on a line.

So far it has been assumed that a line of finite length must have a first and a last point. But Dr. P. K. Menon[36] suggests that this assumption may not be necessary. All that is necessary to suppose is that the points have a lower and an upper limit. Under this assumption, when a line is cut into two, it is not necessary to suppose that the cut must belong to one of the parts. It may belong to neither. We may simply say that it is the upper limit of the points of the first part and the lower limit of the points of the second part. On this view, the difficulty about the beginning of motion disappears. It is not necessary to have a first instant at which motion begins. There may only be an instant which is the upper limit of instants preceding and the lower limit of instants following those at which there is motion.

The Infinite Regress :

That infinity is capable of a precise and consistent formulation shows that an argument or a defintion cannot be regarded as fallacious, simply because it involves an infinite regress. In order to constitute a fallacy an infinite regress must be of a vicious type. Some of the vicious types of infinite regresses have already been discussed as they arose in the course

of discussing other issues. We shall here discuss some more examples.

McTaggart[35] (Vol. I, p. 184) points out that regarding a substance as merely a whole made up of parts invloves an infinite regress of the vicious type. A subtance A has to be described merely as a whole consisting of parts A_1, A_2, etc. Each of these parts in their turn have no other description, but that they contain parts. A_1 for example has no other description but that it consists of $A_{1\cdot1}$, $A_{1\cdot2}$ etc. ad infinitum. Now unless at any stage we can say "this is $A_{1\cdot1}$, etc." or describe $A_{1\cdot1}$, in terms, not involving its parts, our original substance will never have any description, and will be hardly distinguishable from nothing.

The viciousness arises not from the infinity of the regress but from the stipulation itself, that there ought to be no description except in terms of parts. This stipulation cannot be waived merely by postulating an infinity of such descriptions.

In the case of motion, no such trouble arises. We can point out a moving train and say, 'this is motion'. This ostensive description involves no infinite regress. Even the verbal definition does not involve it. When we say that motion consists in an object occupying one position at one time, another at a subsequent time and positions in between at times in between, we have given a sufficient description of it and do not have to mention every position the object occupies, separately. "Persons whose height is above 5 feet" is a sufficicient description and we do not have to mention every person taller than 5 feet, in order to give a definite meaning to it.

Another vicious type of infinite regress arises, if a thing is defined in terms of disjunctions alone. S may be defined as either A or B or C. A in its turn may be defined as A_1, A_2 or A_3. A_1 again may be defined as $A_{1\cdot1}$, $A_{1\cdot2}$ or $A_{1\cdot3}$....etc., with similar definitions of B and C. If at any stage, we do not get a description which cannot be broken up into disjunctions,

the series will be endless and we would be nowhere near describing a thing.

In order to get a genuine description, we should be able to assert some description say $A_{1\cdot1}$ without disjunction. We must be able to say that S is $A_{1\cdot1}$, and not merely that it is either $A_{1\cdot1}$ or $A_{1\cdot2}$....etc. But in this case all the other descriptions are superfluous. After asserting that S is $A_{1\cdot1}$, mentioning $A_{1\cdot2}$ etc., adds nothing, to the assertion.

There are some very important points made by Indian Philosophers, while discussing the infinite regress. The regress, is a fallacy, or a logical defect, and only arguments and definitions can be condemned on the ground that they involve an infinite regress. If this elementary fact is forgotten, we may condemn not arguments or definitions but facts, because they involve an infinite regress. The tree springs from the seed, and the seed in turn springs from another tree. Even if this regress is infinite, it cannot be regarded as vicious. The tree is an observed fact and so is the seed. Both can be defined ostensively by saying "this is the tree, and this is the seed". The regress of the tree and the seed even if infinite is therefore a reality (Pramanik) and not an artifact of logic. Jayanta* would say that, the basis (*mula*) of the regress, *viz.*- the tree is established independently of the regress and the regress is not capable of robbing it of its reality. There is thus no question of a fallacy here.

But on the other hand if the tree, the seed and their causal relation, were not observed facts, and the tree were defined as something which comes out of a seed, and the seed in turn were defined as something, which is found on a tree we would get an infinite regress, which would be an artifact of logic and not a fact of nature. Such a regress would undoubtedly be vicious.

---

*मूलक्षतिकरीमाहुरनवस्थां हि दूषणम् ।
मूलसिद्धौत्वरुच्याऽपि नाऽनवस्था निवार्यते ।[25] (p. 21)

Upshot :

Zeno's arguments on the whole have been shown to maintain that both the infinite and finite divisibility of Space and Time involve difficulties. Some logical problems concerning infinity and the infinite regress have been discussed and the possibility of a consistent conception of infinity and a valid form of infinite regress has been indicated.

the series will be endless and we would be nowhere near describing a thing.

In order to get a genuine description, we should be able to assert some description say $A_{1\cdot1}$ without disjunction. We must be able to say that S is $A_{1\cdot1}$, and not merely that it is either $A_{1\cdot1}$ or $A_{1\cdot2}$....etc. But in this case all the other descriptions are superfluous. After asserting that S is $A_{1\cdot1}$, mentioning $A_{1\cdot2}$ etc., adds nothing, to the assertion.

There are some very important points made by Indian Philosophers, while discussing the infinite regress. The regress, is a fallacy, or a logical defect, and only arguments and definitions can be condemned on the ground that they involve an infinite regress. If this elementary fact is forgotten, we may condemn not arguments or definitions but facts, because they involve an infinite regress. The tree springs from the seed, and the seed in turn springs from another tree. Even if this regress is infinite, it cannot be regarded as vicious. The tree is an observed fact and so is the seed . Both can be defined ostensively by saying "this is the tree, and this is the seed". The regress of the tree and the seed even if infinite is therefore a reality (Pramanik) and not an artifact of logic. Jayanta* would say that, the basis (*mula*) of the regress, *viz.*- the tree is established independently of the regress and the regress is not capable of robbing it of its reality. There is thus no question of a fallacy here.

But on the other hand if the tree, the seed and their causal relation, were not observed facts, and the tree were defined as something which comes out of a seed, and the seed in turn were defined as something, which is found on a tree we would get an infinite regress, which would be an artifact of logic and not a fact of nature. Such a regress would undoubtedly be vicious.

---

*मूलक्षतिकरीमाहुरनवस्थां हि दूषणम् ।
मूलसिद्धौत्वरुच्याऽपि नाऽनवस्था निवार्यते ।[25] (p. 21)

Upshot :

Zeno's arguments on the whole have been shown to maintain that both the infinite and finite divisibility of Space and Time involve difficulties. Some logical problems concerning infinity and the infinite regress have been discussed and the possibility of a consistent conception of infinity and a valid form of infinite regress has been indicated.

XVII

The Reality of Motion

(7) Some more Arguments.

We can now consider some other arguments against motion.

Nagarjuna argues :—

"A goer cannot go, nor can a non-goer go. Who else then can go ?*"

Our answer is 'it is the goer who goes'. "One who goes, goes" is tautologous but not false.

Nagarjuna further adds :—

"Is going the same as the goer, or different? If they are the same, the agent and the action become identical. If they are different, there will be going without a goer and a goer without going. Thus either way motion cannot be explained."**

*गन्ता न गच्छति तावदगन्ता नैव गच्छति ।
अन्यो गन्तुरगन्तुश्च कस्तृतीयो हि गच्छति ?[42] (p. 92)

**यदेव गमनं गन्ता स एव हि भवेद्यदि
एकीभावः प्रसज्येत कर्तुः कर्मण एव च ।(p. 104)
अन्य एव पुनर्गन्ता गतेर्यदि विकल्प्यते
गमनं स्याद्दते गन्तुर्गन्ता स्याद्गमनाद्दते ॥[42] (p. 105)

The argument is based on the assumption that if the terms are different, they cannot be related, or the relation of difference precludes all other relations.

Other arguments of Nagarjuna are more or less similar to these.

There is another argument against motion which finds it difficult to say whether the fish moves first or the water which gives way to the fish, moves first. Suppose a stone is sinking in water. When it touches the surface of water, it cannot move unless it pushes the water out of its way. On the other hand, the water cannot be pushed out of the way unless the stone moves.

The answer is clear. The sequence of events is (1) contact of the stone with water, (2) movement of the water, (3) movement of the stone. The earlier event can take place without the later but not vice versa. It is not true that the water cannot move unless the stone moves. The condition for the movement of the water is *contact* with the unsupported stone, and not the movement of the stone.

When the stone is said to be in contact with the water, we may suppose that there is space between the surface of the water — say *a* — and the surface of the stone — say *b*. If this is not accepted, we shall have to suppose that the stone and the water have a common surface which is both stone and water.

Now there is an argument mentioned by Lotze (not in this connection) in his Metaphysics, which must be considered to allow for the possibility of the stone pushing the water. It runs as follows:—

".....before the motion could transfer itself from the limits of *b* to *a*, it would have to traverse, no matter in how short a time a certain space intermediate between the two, and during this time it would be a state which was the state of nothing"[33] (Vol. II, pp. 31-32).

The difficulty arises from supposing that there is such a thing as *a* 'state of motion'. There is here nothing beyond the successive positions of the stone and the successive positions of water. Contact is said to occur when there is some minimum distance between the stone and the water. The "distance in contact" may not be an absolute notion but only a comparative one like baldness. When the required amount of contact occurs, the water also in addition to the stone begins to change positions. There is no such thing as a "state of motion" which is handed over by the stone to the water, as one hands over money to a shop-keeper.

Lotze argues that if there is no distance between two objects in contact, they cannot act on each other, and therefore action can take place only at a distance. ....."*a* and *b* being already stationed at the same point of space cannot by any attraction be brought nearer, nor could any force of mutual repulsion, however actively manifested in other ways, avail to part them asunder, there being no reason why the initial movement tending to separate should take any one direction rather than another"[33] (Vol. II, p. 29).

This argument only proves that if *a* and *b* have a common part, there cannot be mutual action of the common parts. But the principle that action at a distance is impossible, can have other meanings. It may mean that causal action is continuous, such that if *a* is the cause at a place *a'* and *b* the effect at a place *b'* there must be other effects of *a* at places intermediate between *a'* and *b'*. A real refutation of the principle that action at a distance is impossible lies in the denial of continuity in the causal series.

Bradley in his 'Appearance and Reality' has levelled some arguments against the reality of change and motion. "Something *A* changes and therefore it cannot be permanent. On the other hand, if *A* is not permanent, what is it that changes? It will be no longer *A*, but something else. In other words, let *A* be free from change in time, and it does not change, But let

it contain change, and at once it becomes A_1, A_2, A_3. Then what becomes of *A* and of its change, for we are left with something else? Again we may put the problem thus. The diverse states of *A* must exist within one time and they cannot, because they are successive"[6] (pp. 38-39). "*A* is to have a plurality in succession, and yet simultaneously. This is surely a flat contradiction"[6] (p. 39).

Bradley uses the phrase "within one time" in the sense of at the same time. But a succession of states can also be "within" one time in the sense in which the points on a line are "within one line", or are members of the same series.

Again no contradiction between succession and simultaneity is involved. The phases of *A* must be simultaneous with *A* but not with each other. The relation of succession is between the phases. The relation of simultaneity is between the phases and *A*.

The phrase plurality in succession is vague. It may mean (1) a relation of succession between pluralities, (2) many successions. It is the latter that is required in the present case, in the sense that many relations of succession are involved in the phases of *A*. Any phase except the first has this relation to some other phase.

It is not necessary that the members of a class or parts of a whole must exist simultaneously. If this were so, we would never be able to speak of such a class as all the ancestors of the present Queen of England.

Bradley's complaint that "we are forced to assert that *A* is both continuous and discrete, both successive and present," is therefore unfounded. All we are required to maintain is that *A* is a class or a whole, having many members and the members are related by the relation of succession. It is the members of *A* that are successive, not *A* itself.

The feeling that the members of a class must be simultaneously present, arises from the ambiguous use of the word

"is". The statement 'A_1 *is*' a member of A, says that "A_1 is" and this is interpreted to mean A_1 is present. But this interpretation is wrong.

The word 'is' has at least three meanings (1) a syntactical one, *i. e.* asserting a predicate of a subject. In Sanskrit, this can be dropped and thus creates no confusion between the "present" and the asserted. The assertoric "is" does not denote existence. "A hare's horn is no different from a castle in the air", we can say but the "is" here does not signify that either the horn or the castle exists; (2) 'Is' can mean present, indicating contemporaneousness. This again need not denote existence. A sentence like "Nugudas is a king of the Wadigas" occurring in a novel does not indicate the existence of either Nugudas or of Wadigas. It, however, does indicate contemporaneousness. (3) 'Is' can denote existence as in 'Is there a God ?'

In the proposition "A_1 is a member of A" the membership of A_1 is asserted, not its presence. In case the change in question is real and not fictitious, the "is" also denotes the existence of A_1, but it need not denote its presence unless the contemporaneousness of A_1 is to be asserted.

Upshot :

Some more arguments against motion and change have been examined and found faulty.

XVIII

Logic and Reality

Consistency and Illusion :

The long list of arguments against the reality of Space, Time and Motion raises a problem, fundamental to all philsophy, *viz.*- the problem of the relation of logic to reality. It is assumed that whatever is real can be described, if at all, in non-contradictory terms and whatever is such that it can have none but contradictory descriptions, cannot be real. On this suppostion, if the paradoxes of motion cannot be resolved, it will follow that motion is not real.

But declaring motion to be unreal does not end our troubles. Even if we regard it as unreal, it cannot be denied that it is experienced. It is not therefore unreal in the same sense in which the "son of a barren woman" and other contradictory things are unreal. These contradictory things are never experienced. Calling them, unreal therefore raises no problems. But if we regard motion as unreal, we shall have to say that the experience of motion is an illusion.

Some philosophers think that they can escape from all responsibility of resolving contradictions, if they declare that what gives rise to the contradictions is an illusion. The *Anirvachyavadins*' chief tenet is that the world is indescribable

and therefore its descriptions are bound to be fraught with contradictions. It is a futile task to try to resolve these contradictions, because they are inherent in the very nature of things. Whichever way we describe the world, we contradict ourselves. The world is therefore illusory. This of course is not resolving the contradiction, but simply using another word for describing the contradictory state of affairs.

But the *Anirvachyavadin's* supposed way of escape is not really open to him, because the question remains how even an illusion can be self-contradictory[30·2] (p. 268). Even the illusionist cannot accept the statement "illusion is not an illusion". Illusion therefore can no more be self-contradictory, than reality.

The illusionist can reply that his statement amounts to the following: "*X* is not *X* ∴ *X* is an illusion". This does not imply the statement "illusion is not an illusion".

But this reply is inadequate, because the *X* stands for every thing including illusion and therefore the statement "illusion is not illusion" does follow from the argument. The illusionist can escape from this only by arbitrarily excluding "illusion" from the scope of the meanings of *X*.

Such an exclusion is arbitrary, because the doctrine of "indescribability" (Anirvachyavada) maintains that "all descriptions of reality are self-contradictory". But this last statement itself is a description of the descriptions of reality. The illusionist maintains without any reason that though the descriptions of reality are self-contradictory, the description of these descriptions is not.

But do the descriptions of reality exist or not? If they exist, they are in that sense real and the description of those descriptions would be contradictory.

It may be maintained that just as the phrase "son of a barren woman" describes nothing, the so-called descriptions of reality describe nothing. But why should the descriptions

of nothing be self-contradictory? "The satellite of the moon", "the horns of a hare" etc, are also descriptions of nothing. But they are not self-contradictory? In the same way, even if we suppose that in describing motion we are describing nothing, the possibillty of getting a consistent description cannot be ruled out.

The notion that the idea of illusion, can help in resolving contradictions, I think, arises from dreams. I may dream that I am a king and after waking up find that I am not. This contradiction is resolved by supposing that the dream is an illusion.

But the dream is not an illusion in the sense that it is self-contradictory. A dream can be as consistent as the most realistic of novels. Dreams themselves are not self-contradictory, they only contradict waking experience. A description of *A* may be derived from a dream, and also from waking experience. When these two descriptions are contradictory, we resolve the contradiction by supposing that the description derived from a dream is not a true description.

A more fundamental question can now be raised. Why must the descriptions of reality be self-consistent?

If we restrict ourselves to experienceable reality, descriptions serve at least three purposes (1) transferring the the experience of the speaker to the listener and (2) making the listener familiar with our experience as when a patient describes his pain to the doctor (3) descriptions which seek to reconstruct reality. If *A* is described by the description, then by following the description in practice, we can actually make *A*. By following the description of a shirt given in a book on shirt-making, we can make a shirt.

The first two may be self-contradictory. A poet might say "he is seeing and yet blind, and hearing yet dumb" and succeed in arousing the same emotions in the mind of the reader as he himself is experiencing. If the emotion of the reader and

the emotion of the poet are similar, the description must be regarded as true, though self-contradictory. Similarly if a patient tells the dentist, that he feels a tooth-aching in the place from where the tooth was extracted, this may enable the dentist to understand his trouble. The description again must be regarded as true though self-contradictory.

If the descriptions of motion belong to these two classes, *i. e.* if their only purpose is to transfer our experience of motion to the listener or to make him familiar with that experience, the descriptions may with impunity be self-contradictory, as long as they serve these purposes. In order to feel about motion in the same way as Zeno and Nagarjuna did, or to understand *how* they felt about motion, it might be necessary to bear their self-contradictory descriptions of motion in mind. In such a case, there is no necessity to resolve the contradictions.

But a self-contradictory description will not enable us to construct motion or to deal with it in any other way. The recipe for making a cake cannot be self-contradictory, otherwise no one can make a cake by following it. If a book on cookery says, "take sugar and yet don't take it, mix sugar with flour, but take care not to mix sugar and flour, etc....." some people may find it entertaining or even "profound" reading, but it can hardly enable any one to make a cake.

Scientific descriptions aim at such a statement as will enable us to reconstruct what is described. When a Botanist describes a flower, the purpose of the description is not to make us feel what the Botanist feels on looking at the flower. "And when my heart with pleasure fills, it dances with the daffodils" says Wordsworth, but such a sentence occurring in a Botanical treatise on daffodils will disappoint us. When we pick up a book on Botany, we are not interested in finding out what happens to the heart of the author when he looks at the daffodils. We shall suppose that the Botanical description of the flower has served its purpose, if we can make a daffodil in a laboratory with the help of that description.

It can be argued in this connection that a description which merely enables us to share or understand the experience of the describer, only secondarily gives us knowledge of what is described, whereas descriptions which enable us to reconstruct what is described, give us such knowledge primarily. Surely a man who can make a flower in the laboratory knows the flower, in a truer and deeper sense of knowing, than a person who merely feels in a particular way about the flower. Bergson's contention that intelligence merely enables us to deal with reality, but not to know it, cannot thus be accepted. Ability to deal with and to reconstruct reality is knowledge of reality *par excellence*.

The sheet anchor of anti-intellectualistic philosophies, like that of Bergson, is the unresolved paradoxes which inevitably arise in describing reality. These philosophies argue, that because some contradictions have not yet been resolved, therefore they cannot be resolved and are inherent in intelligence itself.

But this argument is obviously unwarranted. It lies on the shoulders of anti-intellectualist theories to prove, not only that some contradictions are unresolved, but also that they will never be resolved. But this seems to be an impossible task.

"*A* is not *A*" is the form of all contradictions. In order to be sure that the contradiction is final, we must be sure that all the meanings of *A* are known. It is always possible that the *A* of the subject stands for one meaning of *A*, whereas the *A* in not *A* stands for another. If we say "the son of *E* is not the son of *E*", it may be that the first son stands for an adopted son, whereas the second son stands for a real son.

Paradoxical arguments are very often presented in the form of a dilemma, based on the excluded middle. *A* is either *B* or not *B*, we are told. But it is not always easy to be sure that there is no middle, and that *A* cannot be something else, besides *B* and not-*B*.

In the first place, there may be no excluded middle if B is a continuous notion like bald. It is not true that A is either bald or not bald.

Very often the non-exhaustiveness of the alternatives is not apparent without an instance. There is a sort of empiricism in the realm of logic also. Just as one cannot form an idea of red without actually seeing the colour, one cannot conceive many purely logical properties without actually coming across situations in which they are involved. It looks perfectly obvious that any two events, must either be simultaneous, or one must be the later. But the theory of relativity has shown that the alternatives are not exhaustive, and that there is a third possibility, *viz.*- that the two events do not belong to the same time-system. Until the actual examples in which this possibility is realised are considered, it seems extremely unlikely that anyone, however intelligent he is, will regard the alternatives, either simultaneous or successive as anything but exhaustive.

Sometimes a contradiction arises because the ideas alleged to be contradictory are not logically pure. An idea is logically pure if it is not a complex of component ideas which are logically independent. Suppose A is the idea and A_1 A_2 its component ideas. If the scope of the correct application of A_1 is identical with the scope of the correct application of A_2, we may say that A is logically pure. Or if A_1 always implies A_2 and A_2 always implies A_1, we may regard A as logically pure, *i. e.* all it stands for is logically interdependent.

Now A_1 and A_2 may be different descriptions of the same thing, and thus logically equivalent. They appear different because they mention different relations of the same thing. For example "The Prime Minister of India" and "the sone of Motilal Nehru". Or they may be equivalent, *i. e.* mutually substitutable symbols as require and want.

Whether A_1 and A_2 are equivalent symbols can be known by knowing the vocabulary of our language. But it is not so

easy to know whether they are the descriptions of the same thing. One can be reasonably sure that A_1 and A_2 are the descriptions of the same thing in a given set of axioms and rules, but it is difficult to demonstrate that this will be so under all sets.

For example 8 - 5 and 25/7 are both the descriptions of 3 (in a sense) in the scale of 8, but in the scale of 10 the former alone is the description of 3 while the latter is the description of 3·571..... A contradiction may therefore arise if the proposition 8 - 5 = 25/7 occurs in the latter set.

To be sure therefore that a contradiction cannot be resolved is to be sure that all possible sets have been examined.

It cannot be said that a general proof can be given without examining all sets, because the proof itself will be within some set. The question will therefore remain whether it is valid in all possible sets.

An opponent may here argue. "You are trying to prove the proposition that 'it cannot be proved that a contradiction is unresolvable'. Now your proof also operates within a certain system of axioms and rules. How do you then claim for it a generality, which you deny to the proof that a contradiction is unresolvable?"

In order to meet this objection, we must remember a vital distinction. The truth of a proposition may be quite general though it has not been proved by using all systems. "That a proposition P is a theorem in a particular system" may be true quite generally. This statement does not refer to other systems at all and therefore cannot be falsified by other systems. Similarly, we have the following statement :—

"The proposition that 'a contradiction is unresolvable cannot be proved' is a theorem in a particular system." The statement in the inverted commas cannot be falsified by reference to other systems, because it does not refer to other systems at all.

On the other hand, the statement that a particular contradiction is unresolvable does refer to all systems. It asserts that there is no system in which the contradiction is resolved.

It is obvious that such a proposition cannot be established without taking account of all systems.

We may look at the matter in another way. Suppose we try to prove that a particular contradiction cannot be resolved by maintaining either (1) that the proposition that it can be resolved is self-contradictory or (2) that it implies a contradiction. This will be a sort or *reductio ad absurdum.*

But proof by *reductio ad absurdum* is completely inapplicable in this case. The strength of *reductio ad absurdum* lies in that if we don't accept p, we have to accept something absurd, and we avoid accepting anything absurd by accepting p. But when p itself is absurd this refuge is not available. Arguing that if we do not accept the contradiction C_1, we have to accept the contradiction C_2, therefore we should accept C_1 assumes that one contradiction is more acceptable than another.

It may be argued that if unresolvable contradictions cannot be found, proofs by reduction, which show that the false alternatives imply unresolvable contradictions, will cease to be valid. This difficulty disappears, if we remember that a contradiction can be unresolvable within a given set of rules and axioms. As long as the unresolvable contradiction emanates from a hypothesis, there is no need to resolve it. We can simply reject the hypothesis. But when it arises in describing something like motion, which is given to us as an experienced fact, which cannot be argued away, we must seek ways of resolving it and if our axioms and rules do not enable us to do so, we must invent new axioms and rules. To suppose that the contradiction will vanish by uttering the magic word "illusion", is an intellectual escapism.

The Way of Intuition :

This type of intellectual escapism, condemns experience at the bar of Logic. On the other hand, the type of intellectual escapism represented by Bergson condemns logic at the bar of "intuition". Bergson casts his lot with Heraclitus and says

that there is only movement and no mover. The question of motion for him is not ".....how are produced in given *parts* of matter changes of position but, how is effected in the *whole* a change of aspect, 'a *moving*' continuity is given to us in which everything changes and yet remains, whence comes it that we dissociate the two terms, permanence and change, and then represent permanence by *bodies* and change by homogeneous movements in space" [4.1] (p. 260).

Motion then is an indivisible whole and must not be analysed into parts. When we try to do this, we tend to create, a static arrow, that moves, and a static path on which it moves. This static path may further be divided infinitely and thus land us in all the difficulties of Zeno, whose arguments.... "all consist in making time and movement coincide with the line which underlies them in attributing to them the same subdivisions as to the line, in short in treating them like that line" [4.3] (pp. 325-327).

What meaning can Bergson assign to the statement that there is motion but no mover ? When the *Kshanabhangavadin* says that there is no arrow that moves, his statement has a definite meaning. He means that it is not the same arrow that occupies different positions. The arrows at different positions are different though similar. But this cinematographic view can hardly be acceptable to Bergson. I cannot think of a meaning, consistent with the philosophy of Bergson, which could be attributed to the statement.

After declaring the paradoxes of Zeno to be an outcome of analysing motion into a mover and its path, Bergson himself resorts to the same analysis for resolving the paradoxes. He declares movement to be "an undivided fact or a series of undivided facts" and that.... "each of Achilles's steps is a simple indivisible act, and that after a given number of these acts, Achilles will have passed the tortoise"[4.2] (p. 113). This only amounts to saying that motion and derivatively its path is to be analysed into indivisible, and not infinitely divisible parts.

In claiming to provide a view of motion, more acceptable than that of logical analysis, Bergson is only advocating one way of logical analysis against others.

This criticism is valid, not only against Bergson's account of motion, but also against his whole philosophy. According to him, it is intuition and not intellect that enables us to grasp reality. "The very stuff" of this "reality which is perpetual becoming and never something made is 'duration' ". "In the duration in which we *see ourselves acting*, there are dissociated elements, but in the duration in which *we act*, our states melt into each other. Pure duration is what is most removed from externality and least penetrated with externality, a duration in which the past is big with a present absolutely new. But then our will is strained to the utmost, we have to gather up the past which is slipping away, and thrust it whole and undivided into the present. At such moments we truly possess ourselves, but such moments are rare"[4.3] (pp. 210-11).

The glimpses of reality and duration which Bergson gets in these rare moments are described by him in terms which are not different from logical analysis. "A becoming and never something made", "states melting into each other" etc., is as much a logical analysis of the experience as "something made and not becoming" and "states external to each other". Bergson therefore cannot escape from analysis even when he is talking about "pure duration".

The only difference is that in the case of "pure duration" only those who have enjoyed the "rare moments" Bergson talks of, can say whether Bergson's analysis of it is appropriate or not. But in the case of motion any one who can see an arrow in flight, has a right to decide whether a description in terms of "states melting into each other" or "states external to each other" is the more appropriate, and the method of deciding this can be none other than the orthodox method of reasoning and empirical verification. It is difficult to see how Bergson's "intuition" will help us out in this matter.

Upshot :

Whenever descriptions of reality involve logical contradictions, resort to "illusion" is in no way helpful to resolve them. The only wise course is to try alternative methods of analysis until the contradictions are satisfactorily eschewed. The Bergsonian approach is similarly an escapism, which flaunts half-hearted analysis as the deliverances of intuition.

XIX

Satkaryavad (As Applied to Motion) and Teleology

N. B. :—I have modified the arguments of the Satkaryavad so as to be applicable specifically to motion.

A doctrine diametrically opposed to Bergson's creative evolution is the *Satkaryavad* of the Samkhyas. According to this doctrine, an effect is only a manifestation of what is already there in the cause. We are not here concerned with the original doctrine which refers to change in general but only with its implications with regard to motion.

Suppose an object 0 has moved from *a* to *b*. Its position at *b* can be regarded as an effect of some cause. According to the Samkhya doctrine of causation, it will be wrong to suppose that 0 was not at *b* before it came there. In fact it is potentially at all the places where it is supposed to go, what happens when it actually arrives at *b* is that its position at *b* which was unmanifest before becomes manifest.

If we do not make some such supposition, we shall have to suppose, that the cause of 0's position at *b* is its previous absence at *b*. But this is absurd. A previous absence at *b*

cannot be a cause of the present presence at *b*. 0 was previously absent not only from *b*, it was absent from innumerable other places. How is absence from *b*, distinguishable from absence from some other place? There cannot be varieties of absence. If absence from *b*, cannot be distinguished from absence from *C*, there is no reason why this absence should not cause 0's presence at *C*, rather than at *b**.

There is then some positive cause other than 0's absence from *b*, which precedes and causes 0's presence at *b*. But this positive cause cannot cause something which is naught. In fact something which is naught can never come into being. When we say 0's presence at *b* comes into being, we are asserting that 0's presence at *b* is the subject of the act of coming into being. But how can a subject of an act, be non-existent? "0's presence at *b*" must therefore be regarded as existent in some way—let us say unmanifestly—even before 0 arrives at *b*.

The Samkhya further argues that every effect has a material cause†. The jar has clay as its material cause, because it is made out of it. The material cause inheres in the effect, or in other words the relation of an effect to its material cause is that of inherence. The position of 0 at *b* regarded as an effect has a material cause. The material cause obviously is the object 0 and the place *b*. The effect must have a relation of inherence to this material cause. But how can it have any relation unless it exists? What is non-existent cannot have any relations.

It cannot be maintained that the material cause need not have any relation with the effect before it is produced and that the relation is established simultaneously with the production of the effect, because this position throws us back to the previous difficulty that if it were true anything would be produced from anything‡. If the material cause 0 and the place *b*, had

---

*असदकरणात् ।[23] (p. 65)
†उपादानग्रहणात् ।[23] (p. 65)
‡सर्वसम्भवाभावात् ।[23] (p. 65)

no relation with the effect "0's position at *b*", before 0 arrived at *b*, what was the difference between the said effect and anything else like "0's position at *d*", or the "hare's horn", etc. as far as the said material cause was concerned? It was as unrelated to the effect, as to any other non-existent thing such as the hare's horn. Absence of a relation with *r* cannot be different from absence of a relation with *s*. Why does the material cause then, not produce the hare's horn, and produces only the said effect, *viz*.- 0's position at *b*?

Some people think that the notion of power can help us out here. The cause has a power to produce a particular effect and no other. It is because of this power that the effect "0's position at *b*" and no other is produced by its cause. It is not necessary to suppose that the effect itself is present in the cause even before it is produced in manifest form.

But even this notion of power does not enable us to get over the difficulty of a non-existent effect having a relation with its cause. The cause may have the power to produce the effect, but it must have some relation with the effect, and it cannot have any relation with a non-existent effect in spite of its power.

It is better to regard the supposed power* as nothing but the effect itself in an unmanifest form. A moving object is potentially or unmanifestly at all the positions which it occupies during its tenure of existence, but actually or manifestly it is at some particular position at a particular time.

The Samkhya proves the reality of the effect even prior to its production by the general principle, that an effect is identical with its cause**. The general principle is established by the following argument :—

The effect is a property of the cause. In fact when all the effects of a cause are described, we have fully described the

*शक्तस्य शक्यकरणात् ।[23] (p. 65)

**कारणभावाच्च ।[23] (p. 65)

cause. On the other hand, if we drop the mention of a particular effect, the description of the cause is to that extent incomplete. If 0's position at *b* has a cause, this cause cannot be fully described without mentioning 0's position at *b*. The effect thus being only a property of the cause, must exist whenever the cause exists, and since the cause is supposed to exist even before the effect is produced, the effect itself must be held to be existent even before its supposed production. The supposed production is therefore only a manifestation of what was already in existence.

This argument gives rise to the following obvious objections.

A straightforward identification of the cause and effect as alleged by the Samkhya is unjustifiable. When the cause, say the threads, is produced, the effect, say the cloth, is not automatically produced, when the cloth is destroyed the threads are not necessarily destroyed. The statement "I saw the threads" is not identical with the statement "I saw the cloth". Only the cloth can be worn and not the threads. How then can one identify the thread and the cloth ?

In the case of motion the effect, 0's position at *b*, cannot be identified with its cause 0 and the place *b*. Even when 0 and the place, both exist, 0's position at *b*, does not necessarily exist. When 0 ceases to be at *b*, the effect "0's position at *b*" is destroyed, but neither 0 nor the place *b* is destroyed. The statement "I saw 0 and the place *b*" is not identical with the statement "I saw 0 at *b*". The function of "0's position at *b*" is not served by "0 and the place *b*". The existence of water in a well in India and a thirsty mouth in the Arabian desert, is not the same thing as water in the thirsty mouth. How can then we identify the effect "0's position at *b*" with its material cause "0 and the place *b*"?

According to the Samkhya such objections are adequately answered by the notion of manifestation. The difference in the effect and the cause, pointed out by the objector are all due to

the effect being manifest as effect and unmanifest as cause. When the tortoise contracts its limbs, the limbs have no power to move the tortoise. When he expands them they have this power. One does not argue here that the limbs were non-existent before and are newly created when the tortoise stretches himself. The limbs were there all the time, they were unmanifest, when the tortoise had contracted and manifest when it stretched. The limbs in a manifest state have some power, which they do not have when they are unmanifest[23] (p. 248). The function of the effect "water in the thirsty mouth" is different from the function of the cause "water and the thirsty mouth" because, the effect is unmanifest in the latter.

One and the same thing can have different functions. Fire burns as well as illumines. The difference of function therefore cannot establish a non-identity of the functioners.

Comments on Satkaryavad :

The Samkhya doctrine of Satkaryavad and its implication that a moving object is potentially at all the places it can occupy, is not so strange as it appears at first sight. An astronomer can map out the course of a planet, years in advance. This will not be possible, unless its future positions are in some sense potentially given in the present positions.

This sense will be clear, if we take a simple example. If we assume that an object is moving in a straight line at a particular speed, we can say where it will be at any given time. All the positions of the object and their times are potentially given by two of its positions, and their times. This only means that the positions form an order which is completely determinate with reference to two of its terms. More complex orders require more terms to be determinate. A position may be said to exist potentially in the positions which render it completely determinate.

In this sense the case of the potential existence of the position of a moving body in its previous positions is on all

fours with the implicit presence of the conclusion in the premises. But this similarity is not enough to justify the *Satkaryavad* of the *Samkhyas*, because in the case of inference we can say that the premises and the conclusion have a relation whether we notice it or not. A theorem does not become true only after it is proved to be true. It is always true, whether we hit upon its truth or not. But in the case of motion, "a particular position of a moving object" is real only when the object occupies that position. It cannot be said that the object is always at that position; only we fail to notice it. A conclusion can be rightly said to be implicitly present in the premises, because the implicitness only depends on our not noticing its presence. But the position of the moving body cannot be said to be potentially or unmanifestly present in its previous positions, because even after we notice that it is implied by the previous positions, it does not become, actual or manifest. The manifestation is independent of our noticing or not noticing it. It depends, not on our activity, but on the activity of the moving object.

Thus even the Samkhyas have to accept the emergence of something new, *viz.*- manifestation in the process of change. The unmanifest has to become manifest, it cannot be said that it was manifest even before it became manifest.

The famous Samkhya commentator, Vachaspati Mishra* argues that the phrase "emergence of manifestation" is not legitimate. If this phrase is allowed we may as well talk of the "production of production" and involve ourselves in a vicious infinite regress. The production of a jar is not there before the jar is produced, and one might argue that when the jar is produced the production of the jar is also produced. Manifestation like production does not have to be produced and therefore *Satkaryavad* does not involve the production of anything new.

---

* अथासदुत्पद्यते इत्यत्रापि मते केयम् सदुत्पत्तिः, सती, असती वा, ... असती चेत्, तस्या अप्युत्पत्त्यन्तरमित्यनवस्था ।[23] (p. 252)

This defence of Vachaspati cannot be accepted. Vachaspati can legitimately object to a manifestation standing in need of manifestation, but he cannot object to a manifestation standing in need of production. Illumination does not need to be illuminated, but it certainly needs to be produced.

We have pointed out a similarity between the points on a straight line, which are implicit in any two points, and the effect which according to Samkhya is potentially present in the cause. We must now hasten to point out a vital difference in the two cases. The points on a straight line are implicit in any two points, preceding, succeeding, or one preceding and the other succeeding. But the effect potentially exists in a definite cause, not in any cause in a series. This cause, again, unlike the two determining points on a straight line, must precede the effect.

The statement, "the cause precedes the effect" involves the notion of earlier and later. It is necessary at this stage to point out that the relation of earlier and later cannot be analysed in terms of causation itself, as Grunbaum seems to think. According to him if E_1, is the cause and E_2 is the effect, E_1 is earlier than E_2. We know that E_1 is the cause and E_2 the effect because a change in E_1 is associated with a change in E_2, but not vice versa[20·1] (p. 167).

This analysis is not acceptable even if it is known that a change in E_1 is associated with a change in E_2 and not vice versa. This ensures only a certain asymmetry in the relation of E_1 and E_2. But mere asymmetry is not enough to decide as to which is later. That variation in E_2 is not associated with a variation in E_1, may be due to the fact that the relation of E_2 to E_1 is many-one, as that between the fingers of a person and the person. If we take a different person, we automatically get different fingers, but in taking a different finger we are not necessarily taking a different person. Thus Grunbaum's condition is fulfilled, but still there is no relation of later than between the fingers and the person.

One of the implications of *Satkaryavada* is that nothing can intervene between the cause and the effect, since the effect is identical with the cause. In the usual example of the Samkhya, the cause is the contracted limbs of the tortoise and the effect is the out-stretched limbs. The effect is nothing but the cause itself in manifest form. This could not be possible if something intervened between the cause and the effect. Such intervention would mean that the cause did not exist in the intervening period, and the effect is something which comes into being after the cessation of the cause. Under these circumstances, the effect cannot be said to be identical with the cause. Cause and effect must therefore be regarded as contiguous.

If cause and effect are contiguous, motion cannot be continuous in the sense that in the series of positions of a moving, body there is a position between any two positions, however, near we take them. The contracted position is one position of the limbs of a tortoise, and the out-stretched position is another. If the former is the cause of the latter, there cannot be any position of the limbs intermediate between the out-stretched and the contracted. The *Satkaryavad* of the Samkhyas thus leads to the principle that motion is discontinuous.

Satkaryavad can be led into this principle because it commits the initial mistake of regarding a close similarity as identity. The contracted and the out-stretched limbs of the tortoise are very closely similar, but they are not perfectly identical. The out-stretched limbs have qualities like manipulability, etc., which the contracted limbs do not possess. The two cannot be identified straightaway. The process of stretching the limbs is not merely a process of making manifest, what was already present unmanifestly. It is a process creative of new qualities.

Novelty is a matter of degree, and therefore the question "How much transformation a thing must undergo, in order to be called a new thing, and not a continuation of the old?"

is not capable of a definite answer. When a chair loses one leg, it is still called a chair, even if all the four legs are broken, it may still be called a chair, though a broken chair, when this broken chair is thrown into the fire, we may still say that the chair is burning. A chair ceases to be a chair by innumerable stages, and there is no definite limit before which it was a chair, and after which it is not a chair. The contracted limbs of a tortoise, become the out-stretched limbs, by innumerable stages. The difference between them cannot be properly described as merely a difference between the manifest and the unmanifest, however similar they may be.

The fallacy in the Samkhya argument that an effect must exist in order to have any relation with the cause, has been widely discussed in philosophy in various contexts. A relation between terms can be stated even if the terms, are non-existent. The sentences "Hercules was mightier than Hydra", 'Hamlet was in love with Ophilia' etc., state relations between non-existent objects. "A Pigmy is taller than the Liliputians" states a relation between existent and non-existent people. Similarly we can say that the effect "0's position at *b*" has a relation of inherence with "0 and the place *b*" even before 0 arrives at *b*. We can say that a child born to *A's* wife is the son of *A*, even before the child is born.

The Samkhya argument that if both *A* and *B* are non-existent the non-existence of *A* in no way differs from the non-existence of *B*, is correct to some extent. Two misers were once vying with each other about their saving capacity. One said "I have to-day saved ten thousand rupees by not buying a car". Another replied, "I have saved a million rupees by not buying the Kohinoor". In this case the two cases of not buying are in no way different and nobody would credit the second miser with saving more money than the first.

But suppose the two contenders were not the two misers, who could never have bought a car or a diamond in any case, but two princes, one of whom bought the diamond but not the

car, and the other, bought the car but not the diamond. In such a case one is justified in saying that not buying the diamond saved more money, than not buying the car, and the two not-buyings are different.

In the case of motion similarly, "the absence of a moving object 0 from *b*" which precedes its arrival at *b* is not identical with such an absence when 0 never arrives at *b*. As in the case of the budget of the two princes the conditions preceding 0's arrival at *b*, and the conditions which do not result in 0's arrival at *b* differ markedly. There is therefore no danger of anything being produced from anything even if we suppose that the effect is absent before it is produced because the absence of the effect which precedes the effect, and the absence of the effect which does not precede the effect are not identical.

The Samkhya, alleges that unless we regard the effect as present in the cause, we cannot explain why a particular effect requires a particular cause. But even if we suppose that the effect exists in the cause, the question still remains, why a particular effect exists only in a particular cause. Why is almond oil present only in almonds and not in the sand? The supposed merit of *Satkaryavad*, that it explains the orderliness of the course of nature is thus illusory.

Aristotle's Motion :

The doctrine of *Satkaryavad* will not fail to remind the reader, of Aristotle's definition of motion. He defines it as the fulfilment of what exists potentially — namely, of what is alterable qua alterable, alteration of what can be increased and its opposite what can be decreased, increase and decrease of what can come to be and can pass away, of what can be carried along, locomotion[2] (p. 254).

Aristotle's "motion" also includes change. Let us discuss the definition only as applied to motion. According to Aristotle when 0 moves from *A* to *B*, 0 is capable of being at *B*, even when it is at *A*. It is potentially at *B* when it is at *A*. Movement from *A* to *B* is the actualization of the potentiality.

0 is potentially at *B* means 0 will be at *B* at a particular time in future. Such a statement is not possible unless the future course of 0 is already determined. It is not logical to define motion by making definite commitments about a controversial metaphysical issue like determinism. If the statement is neither true nor false now, it cannot be said that 0 is potentially at *B*.

It may be argued that though the statement "0 will be at *B*" is not true or false now, the statement "0 cannot be at *B*" is false now. This is what is meant by "0 is potentially at *B*".

But we cannot regard the statement "0 cannot be at *B*" as false now unless we regard determinism as false. In case 0's future course is already determined the statement "0 cannot be at *B*" can be false now. Thus this definition also errs in the same way, *i. e.* it makes a commitment on the issue of determinism.

Some times it is argued that determinism in a sense has got to be accepted if the excluded middle is indispensable. The statement "*A* will either occur or not occur tomorrow" is true now. Since a statement about the future is true now, the future is determined[2] (p. 46).

The plausibility of this argument rests on the fact that innumerable possibilities are concealed in the phrase "*A* will not occur tomorrow", though it appears as only one possibility. It even includes the possibility that time may come to an end to-day and there may be no tomorrow. We are not even sure that the number of possibilities is finite. If such a state of affairs is to be described as determinate simply because we can describe it in one sentence, the distinction between determinism and indeterminism ceases to have any meaning.

An enthusiastic indeterminist should note here that, "we are not sure that the number of possibilities is not infinite" falls far short of saying that "we are sure they are infinite". The statement therefore cannot be used in favour of indeterminism.

While discussing the reality of time, I supported Broad's view that statements about the future are neither true nor false. This view should not be interpreted as a pronouncement against determinism and predictability. It has already been pointed out that true predictions are an assertion of the relation of implication between the present and the future. Though the future does not exist now, the implication exists now and thus true predictions are possible even if statements about the future are neither true nor false now. Reverting to the example of the solar eclipse which is going to take place in 1999, the prediction can be regarded as true because the present astronomical situation logically implies the eclipse in 1999, though neither 1999 nor the eclipse exists now. The truth of the prediction refers to the reality of the implication and not to the existence of 1999 and the eclipse.

The similarity between Aristotle and the Samkhya does not end with Satkaryavad. Teleology is another common point between them. Aristotle maintains[2] (Physica, Book V, Ch. 6) that every thing has a natural place and natural motion consists in a body going to its natural place. In moving 'naturally' a body is trying to reach its goal as it were. The upward motion of fire and the downward motion of a stone are natural. Aristotle also postulates a God, as a goal to which all this "moves".

For the Samkhya, the goal is the soul and not God. The Prakriti or Primal nature is said to function solely for the sake of the soul.

Many thinkers have a peculiar weakness for teleology. The words natural and unnatural are also very frequently used, with ethical overtones. The natural is also supposed to be good, and the unnatural bad.

But after all what is natural? Aristotle also uses the term unconstrained. That then is natural to a thing, which has not been caused by something outside. But in this sense the

food that gives energy does so unnaturally. One may add the proviso that what is caused from outside should not interfere with the tendencies of what was within. But what is within? If by within is meant whatever is within the physical boundaries of a moving body, it will be difficult to find any motion which is not caused by something outside the body. When an animal moves to pick up a piece of bread, the piece of bread which is the "final cause" of the movement in Aristotle's sense is outside the animal.

It may be said that the natural movement of a body is that movement which it will have, when left to itself. But a body is never left to itself. A piece of wood when 'left to itself' in the space within the earth's gravitational field has a "natural" downward movement, whereas when "left to itself" in water it has a "natural" upward movement. No precise meaning can therefore be attached to the phrases "natural" and "unnatural".

Upshot :

The Samkhya doctrine of Satkaryavad and Aristotle's definition of motion imply that a moving body is potentially at all the places it ever occupies. This doctrine is not defensible in its present form. Even more difficult to defend is Aristotle's teleological account of motion.

XX

Life, Motion and Predictability

It is an ancient and to some extent natural belief that only conscious beings have the capacity to move themselves. This belief needs some logical analysis.

Self-initiated movement is movement whose cause lies within itself. We find that when all the external influences are constant, a stationary stone does not begin to move. But a sparrow, sitting on the stone does. This gives rise to the supposition that the cause of the movement of the sparrow is within the sparrow, whereas the cause of the movement of the stone is not.

But, if in place of the stone we consider an ice-block on a rock, we are not so sure that the sparrow is fundamentally different from the ice-block. The sparrow may have flown in search of water because it felt thirsty. Now the thirst is partly caused by conditions, outside the sparrow. How and when it will feel thirsty depends on the temperature conditions. An ice-block under the same temperature conditions may tumble off automatically after being perched motionless on the rock for some time. The motion of the sparrow and the ice-block are both partly caused by conditions outside them.

It may be maintained that under the same external conditions, the internal states of the sparrow are liable to more

varied and unpredictable changes than the internal states of the ice-block. But this can be attributed to a very great extent to our ignorance of the internal changes in an organism and may not be inherent in their nature. Ignorance also accounts for the unpredictability, not only of the beginning of movement in a living organism, but also of its course after it begins.

The motion of the wind and the planets which are obvious exceptions to the principle that only conscious beings are capable of self-movement, are explained away by animists mainly on the basis of their regularity and predictability. Now, if the movements of a living being are irregular and unpredictable, it may be because (1) we do not know the rule which they follow or because (2) there *is* no rule which they follow. I will here explain the insuperable difficulties involved in maintaining the second alternative.

Suppose we enumerate all the movements which a living being undergoes during his life-time, or in other words we have a complete account of his behaviour, because his behaviour can be regarded as a sequence of movements. In order to suppose that these movements are irregular and unpredictable, we shall have to suppose that their definite order in time is completely accidental, and the movements by themselves possess no properties by means of which we can show that they form a definite and regular pattern.

As soon as the problem is stated in this form most readers will say that the sequence of movements is not completely accidental, but it is nevertheless true that it is not completely predictable.

Not being completely predictable may be more precisely stated by saying that we can predict what type of movement will follow, but we cannot predict what particular movement will follow. But this only pushes the randomness in a narrower field, the logical difficulties involved in proving that the randomness is inherent and not due to our ignorance, are not eschewed thereby.

If a class of terms has order, by knowing the nature of the order and one or more preceding terms we can predict the succeeding terms. We can also do this even if the terms themselves do not form an ordered class, but have an identical relation with terms of an ordered class. For example, the terms in the sequence, 0, 3, 8, 15, 24..... have a relation $n^2 - 1$, respectively to the terms n, in the sequence 1, 2, 3, 4, 5 . . . . ., and the terms in this latter sequence are ordered.

In order to be sure that the sequence of movements of a living organism is not ordered, we must be sure that (1) there is no relation between the movements which is capable of order, and also that (2) the movements are not systematically related to some other ordered class of terms.

Both these requirements amount to a complete knowledge of all the interrelations of the movements, and all the interrelations of all the terms to which the movements are systematically related.

This knowledge is difficult, if not impossible to obtain for the following reasons :—

In the first place even if the terms in a class have no ordered relation, the classes formed out of the terms may have such a relation. For example, suppose we find the following names written in the following order :—

Krishnan, Ayyar, Gokhale, Damle, Kapur, Shobha-Singh.

There seems to be no relation between the persons named which can explain why they are mentioned in this order. But if we take them in classes of twos, we find that there is a definite relation between the classes which justifies their being mentioned in this order. The first two are South-Indians, the next two are Maharashtrians, *i. e.* neither South-Indians nor North-Indians, but something in between, and the last two are North-Indians. Thus the pairs are mentioned in the geographical order starting from the south.

Again even if there is no order between classes, there may be an order between classes of classes. Suppose we find the following words written in the order given :—

Rose, Lotus, Mango, Gwava, Dog, Cat,
Snake, Lizard, American, Indian.

There seems no explanation why rose etc., are mentioned in this order. In order to discover the reason let us form classes out of them, as in the previous example.

The first two are flowers, the second two are fruits, the next two are quadrupeds, the next to these are reptiles and the last two are men.

But the order is not still obvious. Let us therefore form classes of classes.

The first two belong to the vegetable kingdom, the next two to the animal kingdom and the last two to the human. These three classes have a definite biological order.

Just as classes of individuals and classes of classes of individuals can have order, even if the individuals themselves have no order, the relations between individuals can be in order, even if the individuals are not in order. Suppose the following words are written in the order given :—

Bhim, Arjun, Nakul, Duryodhana.

There seems to be no explanation why the persons named are mentioned in this particular order. But if we consider the relations between them, we at once see an order. Thus the first two are real brothers, the second and the third are half-brothers, and the third and fourth are cousins. These three relations are here arranged in the order of nearness. Real brother is a nearer relation than half-brother, and half-brother is nearer than cousin.

Again if there is no order in relations of relations, there may be order in relations of relations of relations :—

For example, suppose the following names are written in

the order given :—

Jhon, Mary, Charles, Henry, James.

There may be no reason why the persons named are written in this particular order, as long we merely consider the persons and their relations.

Now suppose the relations between them are the following : first is the brother of the second, the second is the sister of the third, the third is the cousin brother of the fourth and fourth is the adopted son of the uncle of the fifth, *i. e.* a cousin by adoption.

Still there seems to be no order.

Now let us consider a relation between these relations.

This relation is that of similarity. The relation brother of is similar to the relation sister of, and the relation sister of is similar to the relation cousin of, and the relation real cousin of, is similar to the relation cousin by adoption. These relations of similarity in their turn are related to each other by the relation more and less. The similarity between the relations brother and sister is greater than that between sister and cousin, and the similarity between the relations sister and cousin is in turn greater than the similarity in the relations cousin by birth and cousin by adoption. There is a definite order of similarity here. Thus an order may be found at any stage in the hierarchy of relations of relations.

Now relations of relations and classes may together form a further sphere where order may be found. For example, suppose we find the following words written in the sequence given :—

(1) Ram, (2) Sita, (3) Ram, (4) Laxman, (5) A, (6) B, (7) C, (8) Train, (9), Passenger, (10) Destination, (11) Customer, (12) Money, (13) Commodity, (14) Shopkeeper, (15) Student, (16) Book, (17) Teacher, (18) Examination.

One would be hard put to explain why Ram, etc. are mentioned in this particular order, if one considers only the

properties of the individuals mentioned. Let us therefore consider the relations between them.

(A) 1 and 2 are husband and wife, (B) 3 and 4 are brothers, (C) 5, 6 and 7 are the first three letters of the alphabet, where B is between A and C, (D) 8, 9 and 10 hang together, because the train (8) carries the passenger (9) to his destination (10) (E). Similarly 11, 12, 13 and 14 hang together, because the customer (11) gives the money, (12) for bringing a commodity, (13) to the shopkeeper. (14) (F) In the same way 15, 16, 17 and 18 form a tetrad, since a student (15) studies a book (16) with the help of a teacher (17) to pass an examination (18).

But we still do not know why the individuals and the relations between them are written in this order.

Let us now consider the classes of these relations (1) *A* and *B* are relations between two-terms, (2) *C* and *D* are three-term relations and (3) *E* and *F* are four-term relations.

Now the order is obvious. The class of relations relating a larger number of terms comes later. The relation between the classes of relations, determines the order.

In a nutshell, we have here (1) individuals (2) relations between individuals (3) classes of relations and (4) relations between the classes of relations.

Thus, given a sequence of individuals there are numerous directions in which we may search for an inherent or a derived order in them. In order to prove that the temporal sequence of movements of a living organism has no inherent or derived order, apart from the accidental one that they happened to occur in a particular sequence, we must prove that no order can be found in any of the numerous directions described above. No such general proof seems to be available.

It must be immediately added that the opposite assumption, that the sequence of movements of a living organism must have some order, has no proof either.

Some writers have advanced a fallacious argument to prove that any phenomenon in the universe must have at least a derived order. The argument in essence runs as follows :—

Since we can proceed endlessly, constructing classes and relations of individuals in a given set, there must be an infinite number of such sets possible, and one at least of an infinite number of sets must be an ordered set[14] (p. 83).

There are many fallacies in the argument. In the first place, it merely assumes that the number of sets must be infinite. In the second place, it surreptitiously uses the word infinity in the sense of all and of all types. An infinite number of sets may contain only unordered sets. It can be argued that if we have all possible sets of all kinds, at least one of these sets must be ordered, on the assumption that there is at least one ordered set. But even if there are a million ordered sets, there can be an infinity of other sets which are not ordered.

The argument can be likened to the following :—

"Starting from 2 and adding 2 at every step, we can proceed endlessly and get an infinite number of numbers. Since these numbers are infinite, at least one of them must be an odd number."

The fallacy now at once becomes blatant. There is an infinity of even numbers alone and we cannot infer the existence of odd numbers from the fact that even numbers are infinite.

It is true that if we have a set of all numbers, odd numbers, must be included in them. But an infinity of numbers is not the the same thing as all numbers.

Upshot :

The ancient belief that only living beings are capable of locomotion is not justified. At the most, it can be said that the sequence of movements undergone by a living organism is

less regular than that of the planets. But regularity or order may not always be obvious on the face of phenomena. There are numerous directions in which it can be found.

XXI

Motion as a Category and its Classification

Some philosophers attach great importance to the notion of category. The distinction between a category and an empirical existent is "the distinction between what is pervasive in experience and what is variable and not pervasive"[1] (p. 322). A category is a "pervasive determination" of things.

Alexander regards motion as a category because "....in fact the category of motion is but another expression of the fact that every existent is a piece of space-time....the category we are now dealing with is more properly described as a motion or a space or a time, or by their abstract terms — motion, spatiality, temporality. Every thing is a motion, a space time"[1] (p. 320).

From these utterances it appears that motion is regarded as a category, because it is a "pervasive determination" of things, since everything is a motion, a "space-time". But this remark is open to criticism. "In what sense", it may be asked, "is pleasure a piece of space-time?" Pleasure may be in time, because it arises, endures, and disappears. But in what sense is it in space? It may be that pleasure is possible only in an organism which has spatial dimensions, and it may be that physical pleasures can be localized in the body. But there are

pleasures which cannot be localized, nor do they have any spatial dimensions.

Alexander has developed his idea of space-time by purely logical arguments. The first argument seeks to prove the dependence of time on Space. It runs :—

If time "....were nothing more than bare Time, it would consist of perishing instants....and would contain neither earlier nor later". An observer of events could not be aware of 'earlier' and 'later' "even with the help of memory. For memory cannot tell us that events were connected which have never been together"[1] (p. 45).

This last remark is not very clear. The crucial words are "connected" and "being together". Connected may mean "related". Now two things can be related even if they are not "together" in the sense of being simultaneous or being proximate in space. My tenth ancestor is certainly related to me though he is neither simultaneous nor proximate to me.

Alexander's argument for the dependence of space and time is open to similar objections. He says...."Space taken by itself in its distinctive character of a whole of coexistence has no distinction of parts.....There must therefore be some form of existence, some entity not itself spatial which distinguishes and separates the parts of space. This form of existence is Time"[1] (p. 47).

This last sentence is surreptitious. There are tremendous difficulties in the notion of space as an empty nothing, and distinctive elements are necessary. But these elements need not be necessarily temporal. It is quite possible that these elements differ in non-temporal qualities.

Alexander further tries to prove that the three properties of time, *viz.*- succession, irreversibility and betweenness are made possible only by the three dimensions of space[1] (pp. 50-56). The statement is not very clear but the following seems to be the line of argument.

If time is represented by a line and its instants by the points on the line, then the points corresponding to an instant have to be represented on a line perpendicular to the time-line and intersecting it at t. If we insist on representing these points on the time-line, we shall have to suppose that two successive points p_1 and p_3 can represent the same instant. In such a case a point p_2 in between the p_1 and p_3 will represent an instant which can be said to be both before and after the instant represented by by p_1 and p_2. p_2 is after the instant as represented by p_1 and before the same instant as represented by p_3. This detracts from the irreversibility of time.

Similarly, if we want to maintain that an instant t_2 (represented by p_2), is between the instants t_1 and t_3 (represented by p_1 and p_3 respectively), we represent the instants p_1, p_2 and p_3 also by the points lying on the lines perpendicular to the plane formed by the perpendicular lines of the first paragraph, and intersecting it at p_1, p_2, p_3, etc. Otherwise, a movenment from p_1 to p_3 from above the plane will bypass p_2 and p_2 will no longer be between p_1 and p_3. If the third dimension is included p_2 will be encountered even if the movement from p_1 to p_3 takes place "above" the first plane, since any pair of p_1 and p_3 will have a p_2 which is between them.

I am not sure that this is what Alexander has in mind, but his words are capable of this interpretation. At least the above view in worth examining on its own account.

The whole trouble in the view arises from supposing that "instants" can be repeated. The truth is that many events can take place at the same instant or more precisely there are simultaneous events. The talk of an instant being repeated is as queer as the talk of simultaneity being repeated. Since instants are never repeated, there is no necessity of representing the same instant by two points on the time line, or by points on any other line. Time is best represented by a single line and the points on this line, being arranged by an asymmetrical

transitive relation, secure both the "irreversibility" and "betweenness" of time.

Alexander regards his Space-Time as "equivalent to motion" [1] (p. 320).

The interdependence of Space and Time is so often emphasized that it has become necessary to point out their independence. In the first place, there is no logical necessity that Time and Space must be interdependent. It is possible to conceive of a spaceless but temporal world, and a timeless spatial world. Secondly, we do have a spaceless experience of Time and a timeless experience of Space. When a number of static objects well within the span of attention are perceived no temporal quality is perceived in them, they are parceived purely in space. On the other hand, a tune is heard as a purely temporal process without spatial extension.

The Vaisheshik Category of Karma :

The Vaisheshik philosophy has developed the notion of "Padartha", which is sometimes translated as "category", but the notion of Padartha is not the same as that of category in the sense of Alexander. For the Vaisheshikas, Padartha is anything that can be spoken about and in reducing the universe to the seven non-overlapping Padarthas, they are in a way prescribing a minimum vocabulary for the description of reality. Motion, which is a form of action or Karma is one of these seven Padarthas. The Vaisheshikas define Karma in the following way :—

"Action is that which inheres in one substance, that which possesses no qualities, and that which is an independent cause of combination and separation*".

Stcherbatsky has suggested that the word independent (*Anapeksha*) means "non-relative" and that the Vaisheshik

*एकद्रव्यमगुणं संयोगविभागेष्वनपेक्षकारणमिति कर्मलक्षणम् ।[28] (p. 2)

philosophy therefore wants to assert the reality of absolute motion.

This does not seem to be very likely, because the Vaisheshikas want to contrast action with quality or *Guna* and substance or *Dravya*, in saying that action is independent. They regard quality as dependent, and *Dravya* brings about combination and separation only with the help of action. If *Sapeksha* means relative, it will have to be supposed that the Vaisheshikas regard the qualities, sound, smell, etc. as relative. It will be difficult to find texts to justify such an interpretation.

On the basis of a commentary on the Vaisheshik Darshan, I suggest that the word *Anapeksha* means, "not dependent on what is produced after its own production"*. To illustrate : when I lift my pen from the table and begin to write, the pen is separated from the table and comes in contact with the paper. The cause of this separation and contact is an action. This action is not dependent on what is produced after its emergence *viz.*- contact with the paper. This is what is meant by the independence of action.

But the *Gunas* or qualities are not independent in this sense. When my fingers come in contact with the pen, my hand also comes in contact with the pen. The Vaisheshikas regard "contact" as a quality. The quality contact with the fingers is the cause of the contact of the pen with the hand. Unlike action, this cause is not independent. It depends on the contact of the hand and the pen, which is produced after it is produced. If the contact of the hand and the pen comes to an end, the contact of the fingers and the pen will also come to an end.

The Vaisheshik view that action inheres only in one substance and that it has no qualities is open to criticism. In a relay race, the action inheres in all the runners. Similarly, actions can be one or many, fast or slow, and thus have the

*स्वोत्पत्त्यनन्तरोत्पत्तिकभावभूतानपेक्षम्[28] (p. 24)

Vaisheshik qualities of number (*Sankhya*) and magnitude (*Parimana*).

Classification of Motion :

The Vaisheshik Philosophy classifies* motions as (1) upward, (2) downward, (3) expansion and (4) locomotion.

According to Aristotle, "the motion of things that are moved by something else must proceed in one of four ways.... *viz.*- pulling, pushing, carrying and twirling"[2] (p. 343). He further argues that carrying and twirling can be reduced to pushing and pulling, because that which carries must be either being pushed or pulled or twirled and twirling is nothing but being pushed by one side and pulled by another.

It will be instructive to compare Aristotle's classification with that of other philosophers.

Plato talks of 10 kinds of motion (1) motion on an axis, (2) locomotion, (3) a combination of these, (4) separation, (5) composition, (6) growth, (7) decay, (8) destruction, (9) external and (10) spontaneous. External motion is that "which can set other things in motion but cannot move itself"; and the spontaneous type "is capable of imparting motion to other things under the forms of combination and disintegration, increase and diminution and disintegration; increase and diminution, coming into being and passing away and is able to move itself as well"[51] (p. 97 ff). The meanings of others are quite clear.

Ouspensky has adopted speed as a principle of classification. On this principle, he gets four classes as follows :—

"(1) Slow motion, invisible as motion, for instance the movement of the hour hand of a clock.

(2) Visible motion.

(3) Quick motion, when a point becomes a line, for instance the movement of a smouldering match waved quickly in the dark.

*उत्क्षेपणमवक्षेपणमाकुञ्चनं प्रसारणं गमनमिति कर्माणि[28]। (p. 1)

(4) Motion so quick that it does not leave any visual impression, but produces definite physical effects, for instance the motion of a flying bullet."

"Our eye sees a line, with a certain velocity of motion, a photographic camera will also "see" a line or a streak...." The eye establishes an exact principle of division of velocities. The eye establishes these divisions for itself, on its own level, on its own scale. And this scale may change. What will not change for instance in connection with the distance, what will remain the same on any scale, is first the number of different kinds of motion, there will always be four—and next the inter-relation of the four velocities with their derivatives, *i. e.* with their results or the inter-relations of the four kinds of motion"[43] (p. 434).

This scheme of classification is not based merely on a logical and philosophical analysis, as the previous ones. Its full justification would require a discussion of experimentally verifiable facts and thus does not fall within the scope of this inquiry. It properly belongs to Physics.

Upshot :

Alexander's view that motion is a category and the Vaisheshik notion of activity as a *Padartha* have been examined. Alexander's arguments for the interdependence of Space and Time have been found to be obscure. Some schemes for the classification of motion have also been considered.

XXII

The Perception of Motion

One of the main psychological questions about motion is the way it is perceived.

It is clear at the outset that motion is not a specific datum for perception. The data for perception are specific to the specific sense-organs. Colour cannot be heard and smell cannot be seen. But motion can be known by all the sense-organs. When I see a plane flying I see its motion, when I close my eyes and hear the sound of a jet plane I have a clear perception of the sound coming from afar and travelling past me. The same is true of the fragrance carried by the breeze. If a pencil is moved on my palm, I feel its movement. A similar example could be given in the case of taste, but for the fact that the area of the tongue over which a taste is felt is very small.

In these cases, instead of supposing that it is movement that is seen or heard, it will be more proper to say that, a datum for sight is seen and a datum for ears is heard as moving.

Just as motion is not a specific datum for a specific sense-organ, it is also not a direct datum for perception. This follows from the fact that a moving object must be seen as moving for some distance in some direction. Now distance and direction are not something which can be directly seen like colours. Their

perception is built up on the cues provided by data that are more directly perceived, and by some relevant physiological impulses. An object is seen moving, parallel to the eyes, because it provides different cues, when it is successively at the left, in the centre and to the right of the centre of vision. The centre of the vision consists of cones which are sensitive to colour while the periphery predominates in rods which are not so sensitive. The eyes follow the moving object and the muscular sensations evoked by this movement of the eyes provides further cues. When an object is moving away from us its size appears smaller and smaller, it loses vividness, and its parts can be masked by other objects which are nearer. Thus an object can be seen to be moving in any direction only if it provides such cues peculiar to its successive positions. An observed movement involves some qualitative change in the appearance of the moving object. Motion as a mere change of position without any qualitative change, is an abstraction and not something directly revealed by perception.

It follows that if the cues giving rise to the perception of motion are present, there will be a perception of motion even if no motion is present. Various such illusions of motion are discussed at length in Psychology. The motion perceived in a cinematograph is very well known. When the interval between two exposures of a picture is too short to be noticeable, the observer feels that the picture was continuously present and when it appears at different positions, it is seen as having come to the new position.

In order that this should be possible, it is necessary that the picture or any other stimulus presented must be capable of being apprehended as a single object. For example, in Figure 1, the five dots at the left are presented first and the five dots at the right are presented next. The dots which are encircled

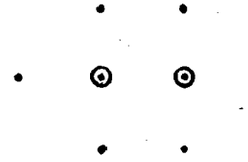

Fig. 1

are presented on both the occasions. The five dots form the figure of a cross, and therefore these successive presentations create the illusion of a moving cross[17] (p. 150). The cross is a definite figure, and when it appears at two places it is possible to suppose that the same cross has moved from one place to another.

The principle becomes clearer by considering another example[17] (p. 151). In Figure 2, the dots *ab* and *ad* are presented first and the dots *ab* and *ac* are presented next. In this case the definite figure which appears in two places is the line formed by the dots. Therefore, it is this line which appears as having moved from one position to another. The angle formed by the line *ab* with the line *ac* or *ad*, does not appear to move because these angles are unequal and are incapable of appearing as an identical figure in two different positions. The line *ab* does not appear to move, because it appears at the same place on successive occasions. Thus a stimulus for being perceived as a moving object must be capable of being perceived as a persistent and definite object with identical features.

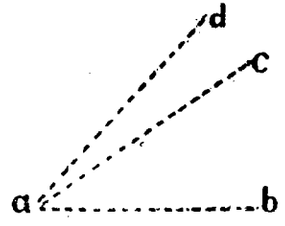

Fig. 2

If the stimulus is a definite object it is not necessary to present it at different places in order to create an illusion of motion. If a circle is quickly exposed and covered alternately or if the light projected on it is made alternately bright and dull, the circle appears as expanding and contracting[17] (p. 174). This is known as the gamma movement. This is perhaps due to the fact that an object is not perceived instantaneously, its central portion is perceived first and gradually its other parts come into vision, and once it is seen it cannot disappear at once, the peripheral parts disappear before the central one.

In all examples of illusory motion described so far, motion was objectively present somewhere, though it was wrongly ascribed. In the cinematograph, there is an actual movement in the projector. Similarly when a figure is exposed or covered, there is an actual movement in the tachistoscope. But sometimes an illusion of motion is created, even when there is no such motion, objectively present. If in a dark room a small point of light is projected on the screen, it appears to move if one looks at it intently[32] (p. 239). A third example is that of perceptual oscillation. When one looks at a white picture of a glass on a black background fixedly, after some time one begins to see an oval window in a black wall. If one continues to look, the first appearance of a white glass against a black background returns again[40] (p. 315). Many pictures demonstrating this sort of effect can be drawn and are mentioned in Psychological literature.

Various explanations of these apparent motions have been offered. Thelin [3] (p. 371) explains the movement of a fixated point on the principle that what is fixated appears as a definite figure and there is a tendency to ascribe movement to definite figures. But everything that appears as a definite figure does not appear to move. The explanation is therefore inadequate.

I think the explanation is to be found in the fact that the span of attention is limited both in time and space. The content of attention cannot be completely identical for more than a very short time. When therefore one tries to attend to a small point, which does not provide enough variety of features for continuous attention, such variety is imposed on it in the form of motion.

Perceptual oscillation may be only a form of the general tendency to oscillate. When we are trying to decide whether to accept a particular job in preference to our present one, we all experience a similar oscillation. For some time, the considerations in favour of acceptance predominate and we feel we

have the decision in favour of acceptance. Immediately, we find that the feeling of decisiveness was illusory and the consideration against acceptance begin to predominate until we feel to be decidedly in favour of rejection. Thus, whenever any situation can be viewed from two points of view, there is a tendency to alternate between these points of view.

Along with oscillation, another usual psychological tendency illustrated by the perception of motion is the tendency to follow the line of least resistance. This is seen to operate in two ways. (1) There is a tendency to perceive familiar objects in their familiar form and to ignore the specific forms given to them by the specific conditions of perception. If we present a lemon in place of a geometrical figure, in the gamma movement experiments, the gamma movement is not seen[17] (p. 176). The lemon is seen as the usual stable object. Subjects who tend to identify geometrical figures with actual objects, instead of seeing them as merely geometrical, do not see the gamma movement. On the other hand, confirmed geometricians who can view a lemon as merely a spherical shape can see gamma movement even in the lemon.

It is clear that perceiving every peculiarity that peculiar conditions of perception impose on an object will make the world appear hazy, unstable and insecure. These peculiarities again do not help in dealing with the objects perceived. The tendency to ignore them has thus a pragmatic justification.

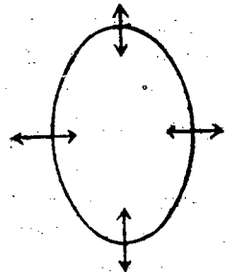

Fig. 3

(2) If an ellipse, as shown in Figure 3 is exposed, the movement is seen to occur in the direction of the arrows. "If a small piece is removed from the centre of sides, a strong movement towards reclosing the figure will result. If a much larger section is taken out (Fig. 4) the two parts act independently and in the direction of flying still farther apart" (p. 176). It is easier to see the gap as filled when it is small, and to see it as expanding when it is large.

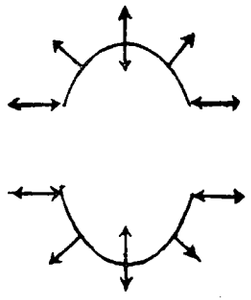

Fig. 4

This is a universal tendency. A man occasionally suffering from disease will try to hide his lapses from full health, whereas if the lapses become the rule and he is constantly ailing he will try to exaggerate and advertise his suffering. In either case, he will keep in line with the strongest tendency in the presented situation.

Perception of motion also exemplifies Weber's principle of proportionality. The difference between the weights of 1 and 2 pounds is felt to be equal to the difference between the weights of 2 and 4 pounds, though Arithmetically the latter difference is twice the former. In the case of motion, a motion may appear as equal to the motion twice faster, if the field in which the faster motion is seen, is twice larger than the field in which the slower one is seen. "In one of these experiments, the movement fields were transposed in a ratio 2 : 1, *i. e.* all the linear measures in one of the moving fields, namely the size of the opening, the diameter of the dots and the distance from one another, were twice as large as the same measures in the other moving field. After the objective velocities were so adjusted that the phenomenal speed in the two movement fields was the same, the objective velocity in the larger field (A) was found to be almost twice as great as in the smaller field[2] (B)" (p. 377).

Another important fact about perceived speeds is that.... ...."the apparent velocity of objects moving objectively with the same velocity is greater for those which move in inhomogeneous than for those which move in relatively homogeneous field"[3] (p. 370).

The exemplification of an analogous principle is seen in the case of the experienced motion of a plane and a train. A passenger in a plane, on looking out does not feel that the plane is moving at a tremendous speed, while the passenger in a train does, because, the air passenger sees only a homogeneous emptyness, with an occasional cloud or a bird, whereas the train-passenger sees, an array of trees, mountains and buildings rushing past. Thus not only an object moving in a heterogeneous field, but the heterogeneous field itself, appears to move faster as compared to a homogeneous field (and thus derivatively the train gives an impression of being faster than the plane).

Vitally connected with the perception of the speed of a moving object is the estimate of the speed of time itself. The same time can be subjectively felt as long or short under different conditions. "In general, a time filled by pleasant, interesting, well motivated activities seems shorter than the time spent simply in waiting. Gulliksen found that subjects who were required to estimate the time elapsing between two signals gave much longer estimates when they passed the time doing nothing or performing a task of monotonous nature than when they were engaged in work.....an interval of 200 seconds was estimated at 242 seconds, on the average, when filled with mere waiting, but as only 169 seconds when devoted to work on problems in long division"[54] (p. 1231).

The comparative estimates of time while doing nothing, and doing work, should not be confused with the comparative estimates of filled and unfilled intervals. "If two clicks mark off the period of silence, the interval is called unfilled. On the other hand a series of five or ten clicks might be presented in which the first and the last clicks would define the interval to

be judged. This would be a filled interval. Filled intervals are perceived as longer than unfilled intervals of the same objective length"[32] (p. 295).

The difference in the two cases lies in this. In judging the filled interval, the subject tends to judge the length of the sequence of clicks, whereas when asked to judge the time between two signals while doing long division, he does not judge the length of the process of long division.

Thus the fact that an inhomogeneous field appears to move faster is not incompatible with the fact that filled intervals are judged to be shorter. If an interval between two clicks is filled by 10 clicks, the individual clicks will follow each other more quickly, than if it is filled with 5 clicks. The sequence of 10 clicks will therefore appear to move faster, inspite of the fact that it is judged as longer.

This creates a dilemma. If monotonous and empty time intervals are like inhomogeneous fields which appear to move slowly, they should, like unfilled intervals, appear shorter. Similarly, if the time intervals which fly quickly because one is occupied, are like the filled intervals, they should be judged as longer.

The dilemma can be resolved only by supposing that it is not merely the number and variety of what passes on in our mind, that determines our perception of time. It will be difficult to maintain that, what passes on in our mind when we are waiting for something and time is hanging on us, is less in number and variety than what passes on when we are happy and time seems to fly.

In addition to variety of content, what is needed for the quick flow of time is the coherence of these contents, and concentration. While waiting, the mind wanders from thought to thought and is not concentrated. Again these different thoughts are not coherent. They may be completely disjointed. On the contrary when one is absorbed in a game of chess, the various

thoughts that pass on in the mind are logically coherent, and the mind is completely concentrated on them.

This is borne out by the following experimental observations. "More disturbing to the judgement of time than the number of events filling an interval is the *meaning* of the material for the listener. The duration of a word, for instance, is judged to be shorter than a noise which actually lasts the same time, and meaningful sentences seem to be shorter than an equivalent series of non-sense syllables"[54] (p. 246).

It is observed that..... "an interval with a striking beginning and end will be perceived as longer than one with indifferent boundaries"[32] (p. 246). This may be due to the fact that "the sharp intense stimuli call particular attention to the passing of time...."[32] (p. 246).

Another important result observed is that...."....a period of idleness and monotony, as in sickness, may seem short when remembered after recovery"[54] (p. 1231). This may be because one tends to think of them as a single experience and does not tend to dwell separately on the different events constituting them. There are not enough such events, nor are they interesting enough for the mind to contemplate them in a longing lingering way.

A person who was sleeping only for 10 minutes may see a dream so eventful that the time required for the occurrence of the events may easily be estimated to be an hour. That events actually seen within 10 minutes are seen as taking the usual time, is an illustration of the principle of the proportionality of movement. If all the objects we see along with ourselves are shrunk proportionately in size, we shall not perceive anything as having become smaller. Similarly when the events which should take an hour are packed within three minutes in a dream, the relative proportions of the events are not disturbed, and therefore their temporal shrinking is not noticed.

For example, a dreamer went to bed when the church clock started striking twelve and woke up when the last of the strokes were heard. During this brief interval, he had a dream which he narrated as under.

"I was serving on a ship. After some days, the ship was wrecked. I saved myself with great difficulty and swam to an island. I spent many days in the hope of being rescued. Ultimately the hope was fruitful, a ship came to the island and I was rescued. I became so influential on the ship that I was chosen to be the captain. I then took to piracy, and navigated many seas. I got tired of that life and went to England, sold the ship and started a business. But one day I was recognised as a pirate, caught and sentenced to death. At the touch of the noose at the gallows I woke up." (Ratcliff : History of Dreams).

The events in the dream are spread over years and yet they are compressed in a few minutes of physical time.

The Primacy of Time :

The study of the perception of space and illusions of movement reveals that space and motion could be perceived in a spaceless and motionless world, provided the cues which give rise to their perceptions were available. The cues themselves are not space or the dimensions of space. Lengths, breadths and depths can be perceived even if there is no length, breadth and depth.

On the other hand, it seems impossible to create an illusion of time. A shorter-time may appear as longer and vice versa, but to experience time when there is no time short or long is not possible.

Some readers may be inclined to argue that even the illusions of space and motion cannot occur without the existence of the brain, and the brain exists in space. The illusions of space are therefore not possible, if there is no space.

This argument entirely misses the point. Even the spatial dimensions of the brain are perceived and it is possible for a

solipsist to argue that their perception arises merely from qualitative cues as in the case of the perception of the third dimension in a picture.

But in the case of time, such solipsism is not possible. In order to create an illusion of time, one event say *A*, will have to be made to appear earlier than another event — say *B* — though it does not *really* occur earlier. But even so, the appearances will have to be regarded as really occurring earlier and later. Even a solipsist cannot deny that one perception occurs earlier than another, and even if he gets rid of everything excepting perception, he cannot get rid of time.

Even if we consider a single perception, it can be analysed into earlier and later portions. Time, unlike space, is not merely a datum for perception, it is a dimension of perception itself. There is both a perception of time as well as a time of perception.

In a spaceless universe, there could be an illusion of space, because illusion need not have spatial dimensions. But in a timeless world, there could not be even an illusion of time, because illusion itself is an event and has temporal dimensions. Time in this sense is more fundamental than space.

The Perception of Force :

From the perception of motion we turn to the perception of force. Commonsense supposes that motion is caused by force. Many thinkers however hold that force is not a scientifically useful concept, nor is it an observed quality. "Do you suppose that the Sun feels muscular effort in keeping the planets in their elliptical paths ?"[7.3] (p. 327) asks Poincare, thereby suggesting that the force that is supposed to cause motion is merely a projection of our subjective feelings on physical phenomena.

Broad observes on this that the sun may not perceive anything because it has no mind. But "the true question is: Does the sun in keeping the earth from moving off at a tangent

have that sort of quality which we who have minds perceive when we forcibly drag a heavy body?"[7.3] (pp. 327-328).

Now there is one vital difference between our "perception" of say colours and our experience of effort. The perception of colour is in no sense coloured or colourful, but the experience of effort can be described as effortful or strenuous, etc. Thus there is less justification for regarding the experience of effort as evidence of the reality of an objective quality called force, than for regarding the perception of colour as evidence for the reality of colour. Colour is not a property of the experience of colour, but force may be a property of the experience of force.

Upshot :

The conditions under which motion is perceived require a well formed object and change in spatial relations either in the stimulus situation or as projected on it. The speed of the flow of time depends on the variety and coherence of the stimuli and the intensity of concentration on them. In experience, Space is less fundamental than Time. The experience of force does not suffice for regarding force as an observable entity.

XXIII

A Retrospect

It is now time to take stock of the outcome of the inquiry. The primary purpose in undertaking the logical and psychological analysis of motion was to attack the main citadel of anti-intellectualist forces. Most formidable logical difficulties have been raised in connection with the motion and its insuperable concomitants, space and time. Some writers take a peculiar delight in raising such difficulties and then try to build a philosophy on such negative foundations. The outcome of philosophies based on such negative foundations cannot be very logical, but I am not inclined to dismiss the work of such writers as mere sophistry. They render a positive service to logic by challenging its complacency and disturbing its dogmatic slumber.

The natural sciences are now too well entrenched to be disturbed by the attacks of the anti-intellectualists. But sciences like Psychology which are trying to adopt the methods of Physics still encounter difficulties in dealing with them. Many Psychologists themselves are not sure that the methods of Physics are applicable in their field. This half-hearted

attitude is likely to affect rigorous research in Psychology. It is, therefore, necessary to show that the extension of scientific method is beset with no special difficulties in Psychology as compared to Physics.

Though it is too much to hope that the solutions suggested in the present work will completely convince the reader, I suppose they will at least convince him that the problems are capable of solution. I am persuaded that the solutions to Zeno's paradoxes have to be sought more or less on the lines of Russell, though I think Russell has not sufficiently realized that Time may present some special difficulties in the way of the solutions offered by him.

With regard to such problems as relativity of motion, the identity of substance, teleology, etc., I am inclined to use Occam's Razor and not to entertain theories which are unnecessary to account for observed facts and do not open up fresh paths for further investigation. With regard to the topics like the perception of motion, I have relied on accepted findings occasionally suggesting new explanations.

In my treatment of Shape, Size and Measurement and of "Life, Motion and Predictability", I have tried to show that the world of Psychology is not fundamentally different from the world of Physics for the purposes of scientific method and that space is not merely physical. It has counterparts in the non-spatial aspects of reality. I hope this will dispose of the prejudice that Psychology does not "belong" in the world of science.

REFERENCES

1. **Alexander, S.** .. (1) Space, Time and Deity. (The Gifford Lectures at Glasgo 1916—1918 in two Volumes. The Humanities Press, New York. First edition 1920, (Reprinted 1927, 1950) Vol. I.

2. **Aristotle** .. (Basic works of Aristotle Edited by Richard Mckeon Random House, New York).

3. **Beardslee, David C. and Westheimer Michael** Readings in Perception. The University Series in Psychology, D. Van Nostrand Company, INC, Princeton, New Jersey.

4. **Bergson, Henri** ..
   1. Matter and Memory. Authorized Translation by Nacy Margaret Paul & W. Scott, Palmer. (London. George Allen and Unwin Ltd. New York : The Macmillan & Co. First published 1911. Reprint 1929).
   2. Time and Free Will. (London, George Allen & Unwin Ltd. First published 1910).
   3. Creative Evolution :—Translated by Arthur Mitchell. First Edition 1911, Reprinted 11, 12, 13, 14, 19, 20, 22, 28, 54.

   The quotation appearing on page 227 is used in 9.2 X and the one on page 229 is used in 48.6, p. 327.

5. **Black Max** .. Problems of Analysis. Philosophical Essays. Cornel University Press, Ithaca, New York, 1954.

6. **Bradley, F. H.** .. Appearance and Reality. Oxford at the Clarendon Press. First Edition 1893.

7. **Broad, C. D.** ..
   1. Scientific Thought, London, Routledge and Kegan Paul Ltd. Broadway House 68—74, Carter Lane E. C. 4.
   2. Note on Achilles and Tortoise (April, 1913).

   (Mind : A Quaterly Review of Psychology and Philosophy, London, Macmillan and Company Ltd., St. Martin's Street W. C. 2).
   3. Perception Physics and Reality. An enquiry into the information that Physical Science can supply about the real. Cambridge : at the University Press, 1914.

8. **Burnet John** .. Early Greek Philosophy.
Fourth Edition, Adam & Charles Black 4, 5, & 6 Soho Square, London, W. I. 1945.

9. **Cajori, Florian** .. 1. The Purpose of Zeno's Arguments on Motion.
Isis Edited by George Santon, Vol. III, 1920-21.

2. The History of Zeno's Arguments on Motion.
Phases in the Development of the theory of Limits. The American Mathematical Monthly, 1915.

10. **Carr, H. Wilden** 1. The Scientific Approach to Philosophy.
Macmillan & Co. Ltd., St. Martin's Street, London, 1924.

2. The General Principle of Relativity in its philosophical and historical aspect, 1922, Publisher 10.1.

11. **Chari, C. T. K.** .. Zeno's Paradoxes reconsidered. The Philosophical Quarterly. Vol. XXVIII, No. 3, Oct., 55. Published by Indian Institute of Philosophy, Amalner, E. Khandesh.

12. **Cheswistek Leon**.. The limits of Science.
(Outline of Logic and of the Methodology of Exact Sciences. London Kegan Paul. Trench, Trunber & Co. Ltd., Broadway House 68-74, Carter Lane E. C. 1948).

13. **Cleugh, M. F.** .. Time and its Importance in Modern Thought.
(Methuen & Co. Ltd., London. First Published in 1937).

14. **Deexit, Shrinivas**.. Manavatawad (Humanism), Philosophy. Special No. January 1952. Navabharat, Published by Sulabha Rashtriya Granthmala, Poona.

15. **Deshpande, D. Y.** 1. McTaggart on Time.
Journal of the Philosophical Association. A Quarterly Review Edited by C. Bhattacharya and D. Y. Deshpande. Printed and Published by B. G. Ogale, B. A., Nakshatra Press, Ogale Road, Nagpur-2. Vol. III, No. 9-10, July-October, 1955.

2. Prof. Ayer on the Past. Mind Vol. LXV 1956, Pub. 7.2.

16. **Eddington, Sir Arthur** .. The Nature of the Physical World Everyman's Library, London, J. M. Dentand & Sons Ltd. Last reprinted 1942.

17. **Ellis, Nlillis D.** .. A Source book of Gestalt Psychology. Prepared. London Kegan Paul, Freach, Trubner & Co. Ltd., Brodway House, Carter Lane, E. C. 1938.

18. **Findlay, J. N.** .. Time : Treatment of some puzzles. Ch. III of Essays on Logic and Language by various authors edited with an introduction by Anthony, Fleu, Bossi and Blackwell, Oxford, 1951.

19. **Fraenkel Abraham, A.** .. Abstract Set Theory 1953, North-Holland Publishing Company, Amsterdam.

20. **Grunbaum Adolf** .. 1. Relativity and Atomicity of Becoming.
The Review of Metaphysics.
A Philosophical Quarterly Vol. IV, No. 2, December 1950 Issue No. 14.
2. A consistent conception of the Extended linear continuum as an aggregate of Unextendel Elements. Philosophy of Science, Vol. XX, Baltimore, Maryland, 1952, pp. 288-306.

21. **Hegel, G. W. F.** .. Lectures on the History of Philosophy. Vol. I, E. S. Haldane. Translated from the German in three Vols. Routledge and Kegan Paul Ltd., Broadway House 68-74, Carter Lane, London, E. C. 4. First published in England in 1892.

22. **Hoskyn, F. P.** .. The Problem of Motion.
Journal of Philosophy, Vol. XXVI, No. 13, June 20, 1929.

23. **Ishwar Krishna** .. Samkhya Karika—with Subodhini & Samkhya Tatwa Kaumudi. First Edition Published by Pandurang Javaji, Nirnaya Sagar Press, Bombay, 1940.

24. **James William** .. 1. Pluralistic Universe, Hibbert Lectures at Manchester College on the Present Situation in Philosophy. Longman Green & Co., 1912.
2. Some Problems of Philosophy. A Beginning of an Introduction to Philosophy. Publisher, 24.1, 30th Street, New York, 1916.

25. **Jayanta** .. Nyaya Manjari, Published by Jaikishandas, Haridas Gupta, Chawkhamba Sanskrit Series Office, Benaras, 1934.

26. **Jones Philip Chapin** .. Discussion, Achilles and the Tortoise. (Mind, October, 1946).

27. **Jourdin Philip E. B.** .. The Flying Arrow : An Anachronism (Mind, January, 1916).

28. **Kanad** .. Vaisheshik Darshan—with Prashastapada Bhashya and Upaskar, Published by Chawkhamba Sanskrit Series Vidyalaya's Press, Benaras, 1923.

29. **King Hugh R.** .. Aristotle and the Paradoxes of Zeno. The Journal of Philosophy, Vol. XVI, No. 21, October 13, 1949.

30. **Lazerowitz, Morris** .. 1. The Structure of Metaphysics. Routledge and Kegan Paul London. First Published 1955.
2. Paper on Paradoxes of Motion read in 52 in Aristotalion Society Proceedings, 51-52.

31. **Lee, H. D. P.** .. Zeno of Elea Cambridge University Press, 1936.

32. **Longfeld, Boring & Weld** .. Foundations of Psychology, 1948. John Wiley and Son, INC., New York.

33. **Lotze, Hermann** .. Metaphysics in three books Ontology, Cosmology and Psychology, in two Volumes. English Translation edited by Bernard Bosanquet. Second Edition Oxford at the Clarendon Press, 1887.

34. **Madhawa** .. Sarvadarshana Samgraha with Hindi Notes, Laxmi Venkateshwar Press, 1925, Kalyan, Bombay.

35. **McTaggart John Ellis** .. The Nature of Existence, Vol. II, Edited by Broad C. D., Cambridge, at the University Press, 1927.

36. **Menon, P. K. D. Sc.** .. In a Personal Conversation.

37. **McTcalf Wimot V.** Notes, Achilles and the Tortoise. (Mind, January, 1942).

38. **Montague, W. M. Pepperell** The Ways of Knowing or the Methods of Philosophy. London George Allen and Unwin Ltd., Ruskin House 40, Museum Street, W. C. 1. New York. First Published 1925. Third impression, 1948.

39. **Morris Charles W.** Has Russell Passed The Tortoise. The Journal of Philosophy, Vol. XXVI, No. 17, August 15, 1959. Published fortnightly by the Journal of Philosophy, INC.

40. **Munn Norman L.** .. Psychology. The Fundamentals of Human Adjustment George G. Harrap and Co. Ltd., 182 High Holborn, London, W. C. 1. First Published 1956.

41. **Murti, T. R. V.** .. The Metaphysical Schools of Buddhism. History of Philosophy Eastern and Western Vol. I. Sponsored by Ministry of Education, Government of India, London George Allen and Unwin Ltd., Ruskin House, Museum Street, First Published 1952.

42. **Nagarjuna** .. Bibliotheca Buddhica IV. Madhyamakvritti MULAMADHYAMIKKARIKAS (MADHYAMIKSUTRAS) avecla Prashnnapada Commentaire de Chandrakirti Public PAR Louis de la Vallee Poussin St. PETERSBOURG 1903, Commissionaires de Academic Imperiele des Sciences.

43. **Ouspensky, P. D.** The New Model of the Universe : Principles of the Psychological Method in its application to problems of Science and Art. Third edition 1938. London Routledge and Kegan Paul Ltd., Broadway House 68-74, Carter Lane, E. C. 4. First edition, 1931.

44. **Pearson Karl** .. The Grammar of Science, Everyman's Library, 1939.

45. **Prabhachandra** .. Prameyakamal Martand, Second Edition, Nirnaya Sagar Press, 1941.

46. **Raju, P. T.** .. Some Reflections on Activism and Zeno's Paradoxes. (The Philosophical Quarterly, April 1955. Vol. XXVIII, No. 1. Publisher 11.).

47. **Ranade, R. D.** .. Shri Gurudeo Ranade Satkar Samiti, Jamkindi Philosophical and Other Essays. Part I. 1956.

48. **Russell Bertrand**.. 1. A Critical Exposition of the Philosophy of Leibniz. First Published 1900. George Allen and Unwin Ltd., Museum Street, London.

2. Human Knowledge, its Scope and Limits. First Published in 1948. Pub. 48.1.

3. Our Knowledge of the External World. As a field for Scientific Method in Philosophy. Fifth Impression. Second Edition. Pub. 48.1.

4. The Principles of Mathematics 1948. Pub. 48.1.

5. A. B. C. of Relativity Revised Edition 1958. Pub. 48.1. (1) Also by the New Museum Library of World, Literature, INC 501, Madison Avenue, New York 22, New York.

49. **Ryle Gilbert** .. Dilemmas. The Tarner Lectures 1950. Cambridge. At the University Press.

50. **Shriharsha** .. Khandanakhanda Khadya, Second Edition, edited by Pandit Vithal Shastri, Scholar, Sanskrit College, Benaras. Printed and published by Bhagawati Prasad, B. A. At the Medical Hall Press, Benaras, for Proprietor E. 7.

51. **Skemp, J. B.** .. The Theory of Motion in Plato's Later Dialogues. Cambridge University Press, 1942.

52. **Stace, W. T.** .. The Philosophy of Hegel. A systematic exposition. Macmillan & Co. Ltd., St. Martin Street, London, 1924.

53. **Steherbatsky, T. H.** .. Buddhist Logic. Vol. 1. 1958 Mouton & Co. S-G Raven Lage, Indo-Iranian Reprints, Edited by the Editorial Board of the Indo-Iranian Journal. IV Originally Published by the Academy of Sciences of the U. S. S. R. Leningrad, 1932, as Vols. XXVI, Part I of the Bibliothica Buddhica.

54. **Stevens** .. A Handbook of Experimental Psychology. Edited by John Weley & Sons, INC, New York. Chapmen & Hall Ltd., London, 1951.

55. **Ushenko Andrew** .. 1. Zeno's Paradoxes. (Mind, April, 1946).

2. The Final Solution of Zeno's Paradox of the Race. The Journal of Philosophy, Vol. XXIX, No. 9, April 28, 1932. Published by the Journal of Philosophy, INC.

56. **Vasubandhu** .. Abhidharmakosha, with a commentary by Rahul Sankratayan. Published by Benaras University, Benaras. Vikram Era 1988.

57. **Ward James** .. Naturalism and Agnosticism. Third Edition (London — Adam & Charles Black, 1906).

58. Whitehead Alfred North .. 1. Process and Reality. An Essay in Cosmology 1927-28. The Humanities Press, New York.

2. Concept of Nature. Tarmer Lectures delivered in Trinity College, Nov., 1919. Cambridge at the University Press, 1930.

3. An Inquiry Concerning the Principles of Natural Knowledge. Publisher 58.2, 1955.

59. Winn, Ralph B. .. On Zeno's Paradox of Motion. Journal of Phil., Vol. XXIX, No. 15, July 21, 1952.

60. Wisdom, J. O. .. Why Achilles does not fail to catch the tortoise. (Mind, Jan., 41).

APPENDIX

A Little About Zeno's Paradoxes

The philosophy of Bergson is one of the most popular among anti-intellectualistic philosophies. The mainstay of this philosophy is the hard logical paradoxes, which according to Bergson are neither resolved nor resolvable. Intellect, in his philosophy, is not only not able to solve but is also responsible for creating them.

The chief among these paradoxes are those concerning infinity, with which philosophers are familiar through Zeno's arguments against motion. The first of these arguments is that a moving body cannot cover a unit distance, for before covering it, it must cover half of it, and before covering half of it, it must cover one fourth of it and so on. Since there is no distance which can be halved to zero and unless the distance is reduced to zero, it cannot be said to be covered, it can never be covered.

Something out of nothing ?

The ancient Indian atomists came across a similar difficulty. It is obvious that every object is made up of smaller parts. Is there then a smallest part, which we may call the atom? Has this atom got parts?

Whether we reply with a yes or no, we land ourselves in difficulties. If the atom has parts, a part of it is smaller

than the whole and thus the atom is not the smallest. If we say that it has no parts, we cannot explain how two atoms together can join to form a bigger size. Two balls can join together to form a bigger size, because a part of one ball touches a part of another. But since the atoms have no parts, a part of one atom cannot touch a part of another atom. The whole of one atom therefore must touch the whole of another. This is a fusion and not a conjunction. Such a fusion cannot result in something bigger than the atom.*

The same difficulty is more usually expressed by saying that if the ultimate parts of a thing have no size, the thing itself cannot have any size. The size of a thing can only be a sum of the sizes of its parts, and the sum of any number of parts of zero size can only be zero. We cannot suppose these ultimate parts to be of a finite size, because a part of a finite size can be further divided and is not ultimate. The ultimate parts can only be given by infinite division. ".....the parts produced by this division can have no size at all, they are end products whose further division is logically impossible"[3] (p. 203).

Now it is true that if a unit is divided into parts such as 1/2, 1/4, 1/8.....etc., the number of parts upto any part of finite size is finite. If therefore the number of parts is to be infinite, there must not be a last part of finite size in our division. From this, it is surreptitiously inferred that the number of parts cannot be infinite unless every part is zero.

Parts of no size a non-sequitur :

Perhaps the idea behind this inference is that the number of *all* and not only some of the parts 1/2, 1/4, 1/8.....etc., is infinite, and we cannot speak of all of a class unless the class

*संयोगश्चाण्वोरण्वन्तरेण सर्वात्मना वा स्यादेकदेशेन वा? सर्वात्मना चेदुपचयानुपपत्तेरणुमात्रत्वप्रसङ्गो दृष्टविपर्ययप्रसङ्गश्च .....। एकदेशेन चेत्सावयवत्वप्रसङ्गः।[4] (p. 232).

has a last member. The class of parts 1/2, 1/4, 1/8.....etc., has no last member.

If we define the class as 1/2, 1/4, 1/8.....etc., together with zero, the class has a last member, *viz.*- zero. Now we can speak of all the members of the class. The number of members in this class is infinite. Thus the number of divisions is not infinite unless the division is "carried" to zero, and if the division is carried to zero, the parts we shall get out of this division will all be equal to zero.

This reasoning contains flaws at every step. It is not at all true that we cannot speak of all of a class unless the class has a last member. We can speak of all the angles of a triangle, but no particular angle can be said to be the last. Further, the phrase "the division is carried to zero" is ambiguous. It may mean that the last division (part) is zero, or it may mean also that the class of parts got by the division contains *all* parts greater than zero.

Now even if (1) the last part is zero, or (2) the class contains all the parts greater than zero, it does not follow that all the parts in the class are equal to zero. Even if the last part is zero, why should this make the first part zero as well? The first part is 1/2 and will remain 1/2 whether the last part is zero or not. A subsequent division does not disturb a prior division, it is only the remainder that is divided. In the second case, no part can be zero since by definition it includes only parts greater than zero.

The Grain and the Mountain :

The principle that even if a distance is divided into infinite number of parts, none of the parts need be zero, solves another difficulty about infinite divisibility, *viz.*- that concerning whole and part. If it is supposed that in an infinite division every division must be zero an inch must be said to be equal to a foot, since both the inch and the foot contain the same

number, *viz.*- infinite of parts of equal size, *viz.*- zero.* But since a body can have infinite number of parts, none of which are of zero size, two bodies cannot be said to be equal simply because they both contain an infinite number of parts. An inch and a foot both contain an infinite number of parts such as

1/2", 1/4", 1/8".....

1/2', 1/4', 1/8' .....

but since every part of the foot is twelve times bigger than the corresponding part of the inch, the foot remains twelve times bigger than the inch.

Another difficulty concerning infinity is that about the beginning of the world. It can be argued, on the lines of Kant both that the world has had a beginning and that it did not have a beginning. If we suppose that the world had a beginning, there must have been an instant at which it began. Now obviously this instant is different from the instants at which it had not begun. But this implies a state of the world before it had begun and therefore the so called beginning of the world is no beginning. The world therefore cannot be said to have a beginning.

On the other hand, if we say that the world did not have a beginning, an infinite number of days must have elapsed till to-day. But an infinite number of days cannot elapse, the infinity of a process consists just in this that it cannot come to an end. An infinity of days could therefore not have passed till to-day and the world must have had a beginning.

Kant's Antinomy Resolved :

This antinomy can be easily resolved. In the first place, instead of talking of the world let us talk of time. The proposition "Time has a beginning" means that there is a first instant

*अनन्तकार्यद्रव्यारब्धत्वाविशेषेण सुमेरुसर्षपयोरपि तुल्यपरिमाणत्वप्रसङ्गात्।[2] (pp. 73-74).

of Time. On this formulation, the phrase "the instant before the first instant" stands for nothing, like the phrase "an integer less than 1". The first instant of Time does not in any way involve anything "before" that instant.

Again it becomes obvious that the world may have had no beginning, if we talk of Time instead of the world. "Time has had no beginning" only means that there is no first instant of Time. The contention that an infinite number of days cannot elapse or come to an end is based on a verbal confusion. It is true that an infinite number of days cannot be counted, because in counting we shall have to start from a first day, and since the number of days is infinite, no day will be the last. But the process of elapsing need not start from a first day, — *i.e.* need not have a start at all — and the series of days even if infinite can have a last day, provided no day is the first, just as the series of negative integers from greater to smaller has a last term, *viz.*- 1, though it does not have any first term. In using the word "elapsing" we are unconsciously likening it to counting which must have both a first term and a last term. The process of counting an infinite number of days cannot come to an end, because in this process there must be a first day as well as a last day, while in the series of infinite number of days, there can either be a first day or a last day but never both. The process of elapsing on the other hand may have only the first day and not the last, or only the last and not the first. Infinite number of days can therefore "elapse" though they cannot be "counted".

Zeno's Dichotomy Resolved :

This view of infinity suggests the solution for Zeno's dichotomy. A moving object successively occupies 1/2th, 1/4th etc., of the distance. This process is likened to the process of counting 1/2th, 1/4th, etc. But this analogy is false. In counting 1/4th comes immediately after 1/2th and 1/8th immediately after 1/4th. On the other hand, a moving object does not come immediately to the place where 3/4th (1/2 + 1/4) of the distance

ends after coming to the place where 1/2 of the distance ends. In fact, there is no place immediately after the place where 1/2 of the distance ends, after 1/2th and before 3/4th there are an infinite number of other distances, *viz.*- 5/8th, 9/16th, etc. Again the process of moving from 1/2 to 1 has a last term, *viz.*- 1. On the other hand, it cannot be the last term of the process of counting 1/2, 1/4.....etc., because in this process every term excepting the first must have an immediately preceding term, and no term immediately precedes 1. Thus the process of counting is fundamentally different from the process of moving and from the fact that one cannot count the distances 1/2th, 1/4th.....etc., it does not follow that one cannot cover these distances by movement.

The Infinite Heads of Hydra :

This very false analogy with counting is present in Prof. Black's [1] comparison of Achilles and Hercules (pp. 109-126). When Hercules cut the head of Hydra another grew in its place, similarly when Achilles comes to the place where the tortoise was, the tortoise has moved to a new position where Achilles must go. In that for every position of Achilles except the last, there is a corresponding and different position of the tortoise, the state of affairs is similar to Hydra having a head corresponding to every cut by Hercules. But there are obvious differences in the two situations which Prof. Black has overlooked. (1) In the run of Achilles, there is no immediately next position after any position, but Hercules cuts the second head of Hydra immediately after cutting the first and so on. Between the first and second head there is no head. (2) In the Hercules there is no last cutting of the last head, but in the Achilles there *is* a last position common to both Achilles and the tortoise. The analogy thus is irrelevant to the essentials of the case.

So far we have discussed the dichotomy and the Achilles which are both based on infinite divisibility. We come now

to the arrow which does not require infinite divisibility. At every instant of its flight the arrow occupies a space equal to itself and occupying a space equal to itself is rest. The arrow therefore is at rest at every instant of its flight, argues Zeno. Owen puts the following in the mouth of Zeno in this connection : ".....if something is true at any and every moment of a period it was true throughout the period"[3] (p. 217).

The Distributive and the Collective :

The fallacy in this argument is obvious. England has had only one person on the throne at any and every year in the last century. From this it does not follow that England has had only one king/queen throughout the last century.

The stadium argument of Zeno is based on the assumption of finite divisibility and is valid under that assumption with suitable provisos. These have been discussed by Russell — in his "Our Knowledge of the External World" and "The Principles of Mathematics". I do not have anything to add to his discussion even by way of exposition.

The outcome of this article can be stated thus—

(1) The plausibility of the dichotomy and the Achilles depends on the confusion between the process of counting and the process of moving.

(2) The arrow argument holds that the motion of the arrow is a series of rests. This is true but does not preclude motion, since the arrow rests at different places and not in the same place.

I do not therefore think that Bergson's "intuition" or "Pure duration" are needed to solve Zeno's paradoxes. The ordinary logical method of analysis and rigorous statement is adequate for the purpose.

REFERENCES

1. Black Max .. Problems of Analysis. Philosophical Essays. Cornel University Press.
2. Keshavamishra .. Tarkabhasha, Edited by S. M. Paranjape. Second Edition, 1917. Published by G. Ramchandra & Co., Budhawar Peth, Poona.
3. Owen, G. E. L. .. Zeno and the Mathematicians. Meeting of the Aristotelian Society at 21, Bedford Square, London, W. C. 1. on 11th March, 1958 at 7.30 P. M.
4. Shankar .. Bhashya on Brahmasutras, Adhyaya, 2, pada 2. (Published by Pandurang Javaji, Nirnaya Sagar Press, Bombay, 1927).

INDEX

By the same Author

English

(1) Abilities and Education
(Available with the Author).

Marathi

(१) कूजन (काव्यसंग्रह) उपलब्ध नाही.

(२) सुखदु:खविवेक ,,

(३) विवेकवाद (तत्त्वज्ञानात्मक लेखांचा संग्रह)
लेखकाजवळ उपलब्ध आहे.

Author's Address
D. P. R. M. Block, New Delhi–1.